

# Collaborative Opportunistic Spectrum Access in the Presence of Multiple Transmitters

Ahmed O. Nasif and Brian L. Mark  
Dept. of Electrical and Computer Engineering  
George Mason University

**Abstract**—We present a collaborative algorithm to enable opportunistic spectrum access for cognitive radios in the presence of multiple co-channel transmitters. A spectrum hole detection and estimation technique based on received signal strength observations is developed, which allows the coexistence of both licensed and unlicensed transmitters. We address the issue of how to perform collaborative spectrum sensing in the presence of multiple co-channel transmitters and how to determine the maximum transmit power that can be used for a given frequency channel by a cognitive radio while avoiding harmful interference to the licensed network. We provide some simulation results to validate the feasibility of our approach.

## I. INTRODUCTION

Recently there has been much interest in cognitive radio (CR) technology and its application to opportunistic spectrum access (OSA) to maximize the utilization of licensed spectrum. Several non-cooperative OSA schemes have been proposed in the literature [1], [2]. The use of collaboration among secondary nodes (CRs) to perform robust spectrum hole detection is discussed in [2]–[4]. The interference analyses given in [2], [5], [6] to quantify the size of the spectrum hole do not readily lead to a simple rule for determining the interference-free secondary transmit power. Most of these schemes do not explicitly account for the presence of multiple co-channel transmitters in the system and the error associated with spectrum sensing.

In [7], we proposed an approach to collaborative spectrum hole detection and estimation based on signal strength (SS) observations obtained by group of secondary nodes with respect to a single primary transmitter. In particular, an approximate expression for the maximum interference-free transmit power (MIFTP) for a single secondary node was obtained. However, in various wireless systems, for example, cellular systems, one must consider the existence of multiple co-channel primary and secondary transmitters. In this work, we propose a scheme for SS-based sensing, localization and collaboration in the presence of multiple co-channel primary and secondary transmitters. Some related work on localization for CR networks can be found in [8], [9]. In our approach, the knowledge of co-channel transmitters can be used along with the raw SS measurements to achieve robustness with respect to co-channel interference. We show that spectrum holes can be identified accurately provided that locally sensed information about

co-channel transmitters is shared among the secondary nodes.

In particular, we propose the maintenance of a distributed database, called the *T-map*, containing co-channel transmitter information including location, power, error estimates, and other information. Using the T-map, a method is proposed to determine the MIFTP that can be allocated to a particular secondary node without causing harmful interference to the existing co-channel primary and secondary nodes.

The remainder of the paper<sup>1</sup> is organized as follows. Section II describes the OSA model in detail. Section III presents a collaborative spectrum sensing scheme to mitigate co-channel interference. An expression for MIFTP in the presence of multiple transmitters is obtained in Section IV. Section V, presents some numerical results to validate the feasibility of our proposed approach. Finally, the paper is concluded in Section VI.

## II. COLLABORATIVE OSA MODEL

### A. SS-based observation model

Consider a group of CRs deployed in the coverage area of a licensed network consisting of multiple primary transmitters operating on a given channel  $\gamma_i \in \Gamma$ , where  $\Gamma$  denotes the band of channels under consideration. We propose a collaborative OSA scheme that identifies the spatial regions where the CRs can reuse the channel  $\gamma_i$ , without causing harmful interference to the primary receivers and to each other. We assume that all transmissions are omnidirectional and the propagation model is homogeneous, with lognormal shadowing. The received SS at node  $i$  due to node  $j$  in dB is denoted by  $R_{ij} = \bar{R}_{ij} + W_{ij} = s_j - g(d_{ij}) + W_{ij}$ , where  $s_j$  is the transmit power of node  $j$ ,  $g(d_{ij})$  is the path loss between two nodes separated by  $d_{ij}$ , and  $W_{ij} \sim \mathcal{N}(0, \sigma_W^2)$ . The net SS received at node  $i$  due to a set of co-channel transmitters  $\mathcal{J}$  in dB is given by  $R_i = 10 \log_{10} \left( \sum_{j \in \mathcal{J}} 10^{\frac{R_{ij}}{10}} \right)$ .

### B. Definition of MIFTP

Denote the set of the co-channel primary transmitters and the secondary nodes by  $\mathcal{P}$  and  $\mathcal{A}$ , respectively. The primary receivers are referred to as *victim* nodes, since they can potentially be disrupted by

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secondary transmissions. The *coverage distance* of primary transmitter  $p \in \mathcal{P}$  is defined as  $d_{\text{cov}}(p) \triangleq g^{-1}(s_p - r_{\min} + \sigma_W Q^{-1}(1 - \varepsilon_{\text{cov}}))$ , where  $s_p$  is the transmit power of  $p$ ,  $\varepsilon_{\text{cov}}$  is a predefined upper limit on the outage probability of an intended receiver located inside the coverage area of  $p$ ,  $r_{\min}$  is the detection threshold of primary receivers (i.e. victims),  $g^{-1}(\cdot)$  denotes the inverse of  $g(\cdot)$  and  $Q(x) \triangleq \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{t^2}{2}} dt$  denotes the standard  $Q$ -function (cf. [7]).

Consider a set of existing co-channel secondary transmitters  $\mathcal{A}_T \subset \mathcal{A}$  and a secondary node  $b \in \mathcal{A} \setminus \mathcal{A}_T$  that is considering to reuse the same channel. Define  $\mathcal{A}_0 \triangleq \mathcal{A}_T \cup \{b\}$ . Denote by  $I_v$ , the aggregate interference power received at a victim node  $v$  due to the transmissions of nodes in  $\mathcal{A}_0$ . The *interference probability* with respect to  $v$  is defined as the probability that  $I_v$  exceeds a predefined given threshold  $i_{\max}$ :  $P_{\text{int}}(\mathcal{A}_0, v) \triangleq P\{I_v \geq i_{\max}\}$ , when all nodes of  $\mathcal{A}_0$  are transmitting. This threshold can be set to satisfy the interference tolerance policy of the primary system.

The objective of the proposed OSA scheme is to quantify the maximum interference-free transmit power (MIFTP) that can be allocated to node  $b$ . The MIFTP for node  $b$  is defined as the maximum power that can be allocated to  $b$  such that the interference probability with respect to any potential victim node within the coverage distance of a transmitter  $t$  does not exceed a threshold  $\varepsilon_{\text{int}} > 0$ , for all  $t \in \mathcal{P} \cup \mathcal{A}_T$ :

$$s_b^* \triangleq \max \{s_b : P_{\text{int}}(\mathcal{A}_0, v) \leq \varepsilon_{\text{int}}, \forall t \in \mathcal{P} \cup \mathcal{A}_T, \forall v : d_{vt} \leq d_{\text{cov}}(t)\}. \quad (1)$$

### C. T-map

In a network consisting of multiple co-channel transmitters, the parameter of interest is  $\Theta \triangleq \{\theta_t, \forall t \in \mathcal{P} \cup \mathcal{A}_T\}$ , with  $\theta_t \triangleq (s_t, x_t, y_t)$ , where  $s_t$  is the transmit power of node  $t$ , located at  $(x_t, y_t)$ . It is clear from (1) that in order to compute the MIFTP, it is necessary to estimate  $\Theta$ . The presence of co-channel interference increases the error in estimating  $\Theta$ , which can be mitigated if the secondary nodes share their estimates with other more distant secondary nodes (see Section III). Therefore, we propose the maintenance of a distributed database, called the *T-map*, containing relevant information about all co-channel transmitters.

In [7], it was shown that given a set of SS measurements, the maximum likelihood (ML) estimator is optimal in the mean-squared-error (MSE) sense and the optimality is achieved as the observation noise becomes vanishingly small. The ML estimate (MLE) of the Cramér-Rao bound (CRB) was found to provide an accurate approximation for the estimation error. Hence, we propose that the T-map store the MLEs of each transmitter's parameters and the associated CRBs:

$$\mathbf{T} \triangleq \left\{ \left( \hat{\theta}_p, \hat{\mathbf{J}}_{\theta_p}^{-1} \right), \forall p \in \mathcal{P} \right\} \cup \left\{ \theta_a, \forall a \in \mathcal{A}_T \right\}. \quad (2)$$

In general, the true parameters of some nodes in  $\mathcal{A}_T$  may not be known. In this case, we treat these particular secondary transmitters as primary transmitters and

estimate their corresponding unknown parameters. For a given frequency channel  $\gamma_i$  and time  $t_j$ , the T-map  $\mathbf{T}_{i,j}$  characterizes the spatial region where secondary transmissions can be allowed. For a static set of primary transmitters, the T-map maintained by a secondary node should converge after a certain period. In a dynamic scenario, the T-map should track changes that take place in the spectrum occupancy profile over time.

## III. COLLABORATIVE SENSING SCHEME

To estimate the MIFTP, the secondary nodes must first update the T-map from their received SS measurements. This SS observation set is denoted by  $\mathcal{O} \triangleq \{(R_a, \mathbf{L}_a) : a \in \mathcal{A}\}$ , where  $R_a$  is the net SS received due to all co-channel transmitters at the secondary node  $a$ , located at  $\mathbf{L}_a \triangleq (x_a, y_a)$ . Since the total number of co-channel primary transmitters,  $M$ , is not known a priori, it is difficult to perform localization of all the primary transmitters simply using  $\mathcal{O}$ .

In the proposed collaborative sensing scheme, the secondary nodes localize the primary transmitters in their local vicinity. Through collaborative information sharing, the T-map can be constructed and maintained in a distributed fashion by means of networking protocols. The SS measurements are shared *locally* among neighboring nodes and localization estimates are shared *globally* via the T-map construct. The presence of co-channel interference from the primary transmitters introduces error in the SS measurements. For example, to localize node  $p$ , instead of  $\{R_{ap} : a \in \mathcal{A}\}$  only  $\{R_a : a \in \mathcal{A}\}$  can be observed, resulting in higher estimation error. This error increase can be mitigated by sharing the estimates of the interfering primary transmitters among the secondary nodes via the T-map and by accounting for the associated co-channel interference. In the remainder of this section, we consider the case  $M = 2$ . Generalization of the approach to arbitrary  $M$  is straightforward.

### A. With no information

Given a set of independent local observations  $\mathcal{O}_1 \triangleq \{(R_a, \mathbf{L}_a) : a \in \mathcal{A}_1 \subset \mathcal{A}\}$  in the vicinity of a primary transmitter, say  $p_1 \in \mathcal{P}$ , the MLE of the parameter  $\theta_1 \triangleq [s_{p_1} \ x_{p_1} \ y_{p_1}]^T$  can be found. In the absence of any information about other co-channel transmitters, the log-likelihood function has the following form [7]:  $F_{1A} \triangleq \sum_{a \in \mathcal{A}_1} \ln f_{R_a | \theta_1}$ , where  $R_a | \theta_1 \sim \mathcal{N}(s_{p_1} - g(d_{ap_1}), \sigma_W^2)$ . The MLE is found by solving the optimization problem  $\hat{\theta}_{1A} = \arg \max_{\theta_1} \{F_{1A}\}$ .

### B. With true information

If the *true* parameter  $\theta_2 \triangleq [s_{p_2} \ x_{p_2} \ y_{p_2}]^T$ , of another co-channel transmitter  $p_2$ , is known, the observations in  $\mathcal{O}_1$  can be modeled as

$$R_a = 10 \log_{10} \left( 10^{\frac{R_{ap_1}}{10}} + 10^{\frac{R_{ap_2}}{10}} \right), \quad (3)$$

where  $R_{ap_i} = \bar{R}_{ap_i} + W_{ap_i}$ ,  $\bar{R}_{ap_i} \triangleq s_{p_i} - g(d_{ap_i})$ ,  $i = 1, 2$ , and  $a \in \mathcal{A}_1$ . Approximating the sum of independent

lognormal random variables by another lognormal [10], yields  $R_a|\theta_1, \theta_2 \sim \mathcal{N}\left(\frac{\mu_{Ba}}{c}, \frac{\sigma_{Ba}^2}{c^2}\right)$ , where

$$\mu_{Ba} \triangleq \ln(k_1) - \frac{\sigma_{Ba}^2}{2}, \quad \sigma_{Ba}^2 \triangleq \ln\left(1 + \frac{k_2^2}{k_1^2}\right), \quad (4)$$

$$k_1 \triangleq e^{\frac{c^2 \sigma_W^2}{2}} \left( e^{c\bar{R}_{ap1}} + e^{c\bar{R}_{ap2}} \right), \quad (5)$$

$$k_2^2 \triangleq e^{c^2 \sigma_W^2} \left( e^{2c\bar{R}_{ap1}} + e^{2c\bar{R}_{ap2}} \right), \quad (6)$$

and  $c \triangleq \frac{\ln 10}{10}$ . Note that  $\bar{R}_{ap2}$  is known and  $\theta_1$  is the only unknown. The log-likelihood function is

$$F_{1B} \triangleq \sum_{a \in \mathcal{A}_1} \ln f_{R_a|\theta_1, \theta_2}, \quad (7)$$

and the ML solution is given by  $\hat{\theta}_{1B} = \arg \max_{\theta_1} \{F_{1B}\}$ .

We have observed that in the region of practical interest,  $\bar{R}_{ap_i} \in [-150, 100]$  dB,  $\sigma_{Ba} \approx c\sigma_W, \forall a$ . If the observations  $\{R_a\}$  are scaled as  $\{\tilde{R}_a\} = \{cR_a\}$ , this approximation can be used to obtain an equivalent but simpler objective function compared to  $F_{1B}$ . In this case, we have  $\tilde{F}_{1B} \triangleq \sum_{a \in \mathcal{A}_1} \ln f_{\tilde{R}_a|\theta_1, \theta_2}$ , where

$$\tilde{R}_a|\theta_1, \theta_2 \sim \mathcal{N}\left(\ln\left(\sum_{i=1,2} e^{c\bar{R}_{ap_i}}\right), c^2 \sigma_W^2\right). \quad (8)$$

### C. With estimated information

A more practical scenario is when only the *estimated* information about other co-channel transmitters is available, via the distributed maintenance of the T-map. Assume that the ML estimated parameters  $(\hat{\theta}_2, \hat{\mathbf{J}}_{\theta_2}^{-1})$  of transmitter  $p_2$  are known. Note that here  $\hat{\theta}_2$  is found by solving the likelihood function  $F_{2A}$  corresponding to  $p_2$  (similar to  $F_{1A}$ ), i.e.,  $\hat{\theta}_2 \equiv \hat{\theta}_{2A}$  and  $\hat{\mathbf{J}}_{\theta_2}^{-1} \equiv \hat{\mathbf{J}}_{\theta_{2A}}^{-1}$ . Instead of  $\bar{R}_{ap2}$ , we can obtain  $\hat{R}_{ap2}$ , where  $\hat{R}_{ap2} \triangleq \hat{S}_{p2} - g(\hat{D}_{ap2})$  denotes the MLE of  $\bar{R}_{ap2}$  and  $\hat{\theta}_2 = [\hat{S}_{p2} \ \hat{X}_{p2} \ \hat{Y}_{p2}]^T$  denotes the MLE of  $\theta_2$ , via the invariance principle (cf. [11, p. 217]), which states that the MLE of a function  $q(\cdot)$  of  $\Phi$  is given by  $q(\hat{\Phi})$ , where  $\hat{\Phi}$  denotes the MLE of  $\Phi$ . Since the MLE of the CRB approaches the estimation error as  $\sigma_W \rightarrow 0$  [12], we propose the following model for  $R_{ap2}$ :

$$R_{ap2} = \bar{R}_{ap2} + W_{ap2} \triangleq \hat{R}_{ap2} + W_{2a} + W_{ap2}, \quad (9)$$

where  $W_{2a} \sim \mathcal{N}(0, \sigma_{2a}^2)$  with  $\sigma_{2a}^2 \triangleq \hat{\mathbf{H}}_a^T \hat{\mathbf{J}}_{\theta_2}^{-1} \hat{\mathbf{H}}_a$ . Again,  $\hat{\mathbf{H}}_a$  is the MLE of  $\mathbf{H}_a$  where

$$\mathbf{H}_a \triangleq \left[ \frac{\partial \bar{R}_{ap2}}{\partial x_{p2}}, \frac{\partial \bar{R}_{ap2}}{\partial y_{p2}}, \frac{\partial \bar{R}_{ap2}}{\partial s_{p2}} \right]^T \quad (10)$$

$$= [\dot{g}(d_{ap2}) \cos \phi_{ap2}, \dot{g}(d_{ap2}) \sin \phi_{ap2}, 1]^T, \quad (11)$$

and  $\dot{g}(d) \triangleq \frac{\partial g(d)}{\partial d}$ .

Following the approach outlined in Section III-B, we can use the scaled observations  $\{\tilde{R}_a\}$  to solve  $\hat{\theta}_{1C} = \arg \max_{\theta_1} \{F_{1C}\}$ , where  $F_{1C} \triangleq \sum_{a \in \mathcal{A}_1} \ln f_{\tilde{R}_a|\theta_1, \hat{\theta}_2}$ , and

$$\tilde{R}_a|\theta_1, \hat{\theta}_2 \sim \mathcal{N}\left(\ln\left(e^{c\bar{R}_{ap1}} + e^{c\hat{R}_{ap2} + \frac{c^2 \sigma_{2a}^2}{2}}\right), c^2 \sigma_W^2\right).$$

Note that  $\hat{\theta}_{1C} \rightarrow \hat{\theta}_{1B}$ , as  $\sigma_{2a} \rightarrow 0, \forall a$ . Our hypothesis is that  $\hat{\theta}_{1B}$  and  $\hat{\theta}_{1C}$  are better estimators than  $\hat{\theta}_{1A}$  in terms of mitigating the error induced by co-channel interference.

## IV. MIFTP ESTIMATION

Suppose we need to calculate the MIFTP for the secondary node  $b \in \mathcal{A} \setminus \mathcal{A}_T$ , (cf. (1)). The true MIFTP as defined in (1) cannot be calculated directly, since the true parameters required ( $\theta_t$ ) are unknown and we only know the MLE  $(\hat{\theta}_t, \hat{\mathbf{J}}_{\theta_t}^{-1})$ ,  $\forall t \in \mathcal{P}$ . For a particular  $t \in \mathcal{P}$ , the *critical distance estimate* with respect to node  $b$  is defined as  $\hat{D}_b(t) \triangleq \hat{D}_{tb} - \hat{D}_{\text{cov}}(t)$ , where  $\hat{D}_{tb}$  and  $\hat{D}_{\text{cov}}(t)$  denote the MLE of  $d_{tb}$  and  $d_{\text{cov}}(t)$ , respectively;  $\hat{\mathbf{J}}_{tb}^{-1}$  denotes the MLE of the CRB corresponding to the error in estimating  $\hat{D}_b(t)$ , [12, Proposition 6].

Let  $\varepsilon \in (0, 1)$  represent the confidence that for a particular realization, the error will be smaller than the corresponding realization of  $\hat{D}_b(t)$ , which must hold for  $s_b^* > -\infty$  [12]. Similarly, for a particular  $a \in \mathcal{A}_T$ , define the *critical distance* with respect to node  $b$  as  $d_b(a) \triangleq d_{ab} - d_{\text{cov}}(a)$ . Define two sets:

$$\mathcal{T}_1 \triangleq \left\{ t \in \mathcal{P} : \hat{D}_b(t) \leq \sqrt{\hat{\mathbf{J}}_{tb}^{-1}} Q^{-1}\left(\frac{1-\varepsilon}{2}\right) \right\},$$

$$\mathcal{T}_2 \triangleq \{a \in \mathcal{A}_T : d_b(a) \leq 0\}.$$

The *minimum critical distance estimate* for node  $b$  is defined as

$$\hat{D}_b^* \triangleq \min \left\{ \{\hat{D}_b(t) : t \in \mathcal{P}\}, \{d_b(a) : a \in \mathcal{A}_T\} \right\}, \quad (12)$$

if both  $\mathcal{T}_1$  and  $\mathcal{T}_2$  are empty; otherwise,  $\hat{D}_b^* \triangleq 0$ . The transmitter  $t^*$  corresponding to  $\hat{D}_b^*$  is called the *critical transmitter* and the *worst-case victim*  $v^*$  is located at  $(x_{v^*}, y_{v^*})$ , where

$$x_{v^*} = x_{t^*} + d_{\text{cov}}(t^*) \cos \phi_{bt^*}, \quad y_{v^*} = y_{t^*} + d_{\text{cov}}(t^*) \sin \phi_{bt^*},$$

$$\phi_{bt^*} \triangleq \tan^{-1} \left( \frac{y_b - y_{t^*}}{x_b - x_{t^*}} \right).$$

Define

$$L_T \triangleq \sum_{a \in \mathcal{A}_T} \frac{e^{cs_a}}{e^{cg(d_{v^*a})}}, \quad h \triangleq \ln \left( \frac{e^{cs_b}}{e^{cg(d_b^*)}} + L_T \right),$$

$$\gamma_2(s_b) \triangleq \frac{1}{c} \ln \left( \exp \left( ci_{\max} - \sqrt{c^2 \sigma_W^2 + \hat{\mathbf{J}}_h^{-1}} \right) \right. \\ \left. \cdot Q^{-1}(\varepsilon_{\text{int}}) - \hat{L}_T \right) + g(\hat{D}_b^*) - s_b, \quad (13)$$

where<sup>2</sup>

$$\mathbf{J}_h^{-1} \triangleq \mathbf{1}_{\mathcal{P}}(t^*) \mathbf{H}_0^T \mathbf{J}_{\theta_{t^*}}^{-1} \mathbf{H}_0, \quad (14)$$

$$\mathbf{H}_0 \triangleq \left[ \frac{\partial h}{\partial x_{t^*}}, \frac{\partial h}{\partial y_{t^*}}, \frac{\partial h}{\partial s_{t^*}} \right]^T, \quad (15)$$

$$\phi_{v^*a} \triangleq \tan^{-1} \left( \frac{y_{v^*} - y_a}{x_{v^*} - x_a} \right), \quad (16)$$

and  $\mathbf{J}_{t^*}^{-1}$  is the CRB of  $\theta_{t^*}$ .

<sup>2</sup> $\mathbf{1}_{\mathcal{P}}(t)$  denotes the indicator function over the set  $\mathcal{P}$ .

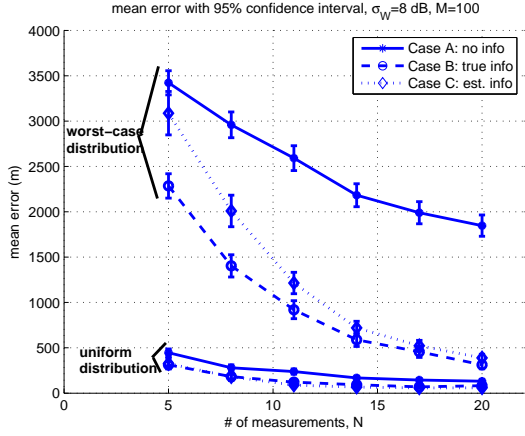


Fig. 1. Localization error  $\mathcal{E}_1$  vs. number of measurements.

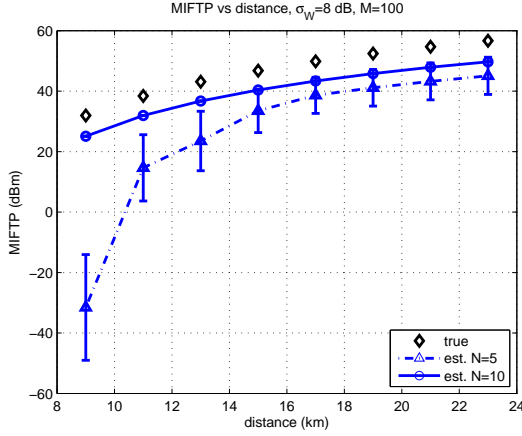


Fig. 2. MIFTP vs. distance from other transmitters.

In the expression for  $\gamma_2(s_b)$ ,  $\hat{J}_h^{-1}$  and  $\hat{L}_T$  denote the MLE of  $J_h^{-1}$  defined in (14)-(16) and  $L_T$ , respectively. Recall that since  $h$  is a function of  $\theta_{t^*}$ , we can invoke the invariance principle to get  $\hat{h}$  in a straight forward manner. For a closed-form expression of  $J_{t^*}^{-1}$  and its achievability conditions, see [7]. Note that  $J_h^{-1}$  implicitly contains  $s_b$  if  $t^* \in \mathcal{P}$ . We approximate the MIFTP as follows: If there exists a  $\hat{D}_b^* > 0$ , then an approximation  $\hat{s}_b^*$  to the true MIFTP  $s_b^*$  is found by solving  $\gamma_2(s_b) = 0$ . The derivation of this result is omitted due to lack of space (cf. [13]). This equation cannot be solved in closed-form if  $t^* \in \mathcal{P}$  and we need to resort to numerical methods. If no root of  $\gamma_2(s_b) = 0$  exists or a root renders the argument of  $\ln(\cdot)$  in (13) negative, then  $\hat{s}_b^* = -\infty$ , i.e., node  $b$  should not attempt to transmit at all on this channel. The effect of localization accuracy is incorporated into MIFTP computation via the CRB term,  $J_h^{-1}$ . In particular, as the estimation error increases, the MIFTP becomes more conservative, ensuring that the interference tolerance threshold,  $i_{\max}$  is met (cf. (13)), but also making the OSA scheme less efficient.

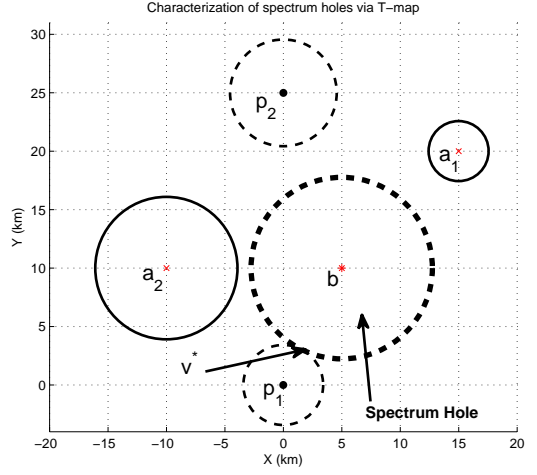


Fig. 3. Pictorial illustration of a T-map.

## V. NUMERICAL RESULTS

For the numerical results presented in this section, we choose system parameter values that reflect the application of OSA to digital TV broadcast bands. The SS measurements are generated using the generic path loss function  $g(d) = 10\epsilon \log_{10}(d)$ , where  $d$  is distance and  $\epsilon$  is the path loss exponent. Each measurement is generated by averaging over 100 raw measurements to reduce the effect of shadowing noise. All secondary nodes detecting the signal of a particular primary transmitter  $p$ , must be located within the *detection radius*  $d_{\det}(p)$ , defined as  $d_{\det}(p) = g^{-1}(s_p - r_a + \sigma_W Q^{-1}(1 - \epsilon_{\text{cov}}))$ , where  $s_p$  is the transmit power of  $p$  and  $r_a$  is the detection threshold of the secondary nodes.

We fix the values of the following parameters: detection threshold for victims  $r_{\min} = -85$  dBm, detection threshold for secondary nodes  $r_a = -90$  dBm, interference tolerance threshold  $i_{\max} = -100$  dBm, outage probability upper limit  $\epsilon_{\text{cov}} = 0.01$ , allowable interference probability upper limit to victims  $\epsilon_{\text{int}} = 0.01$ , shadowing standard deviation  $\sigma_W = 8$  dB and path loss exponent  $\epsilon = 4$ . For these parameter values,  $d_{\text{cov}}(p_1) = d_{\text{cov}}(p_2) = 4.6$  km and  $d_{\det}(p_1) = d_{\det}(p_2) = 6.1$  km.

Consider two co-channel primary transmitters  $p_1$  and  $p_2$  located at  $(8, 0)$ [km] and  $(0, 0)$ [km], respectively, with equal transmit powers of 80 dBm. We are interested in estimating  $\theta_1 = [s_{p_1} \ x_{p_1} \ y_{p_1}]^T$ . To evaluate the performance of our proposed scheme, we find the ML solutions corresponding to the likelihood functions  $F_{1A}$ ,  $F_{1B}$  and  $F_{1C}$  from Section III. As a performance measure we calculate the *mean missed distance* (m),  $\mathcal{E}_1 \triangleq \frac{1}{K} \sum_{i=1}^K \sqrt{(\hat{X}_{p_1}(i) - x_{p_1})^2 + (\hat{Y}_{p_1}(i) - y_{p_1})^2}$ , over  $K = 1000$  independent trials.

In Fig. 1, we plot  $\mathcal{E}_1$  as a function of the number of measurements, along with the corresponding 95% confidence interval. The bottom three curves correspond to measurements taken by secondary nodes located uni-

formly inside the circle with radius  $d_{\text{det}}(p_1)$  centered at  $p_1$ . We observe that although the difference between Cases B and C are negligible, both cases show some improvement ( $\geq 50$  m) over Case A. The top three curves correspond to the worst-case scenario where the measurements are taken by secondary nodes located only at the intersection of the detection regions  $d_{\text{det}}(p_1)$  and  $d_{\text{det}}(p_2)$ . A significant accuracy improvement is seen in Cases B and C ( $\geq 335$  m), more so in B than in C, over Case A. The improvement for the worst-case scenario is much greater because the proposed *compensation* becomes more prominent when both transmitters contribute approximately equally to the measurements.

Consider two primary transmitters  $p_1$  and  $p_2$ , located at  $(0, 0)$ [km] and  $(0, 100)$ [km], and a secondary transmitter  $a$  located at  $(100, 100)$ [km], all transmitting on the same channel with power 80 dBm. We wish to compute the MIFTP of another secondary node  $b$ , using the approach proposed in Section IV. To observe the effect of MIFTP as a function of distance from other co-channel transmitters,  $p_1$ ,  $p_2$  and  $a$ , we vary the location of  $b$  from  $(9, 0)$ [km] to  $(23, 0)$ [km]. The true and estimated MIFTP are found by solving the equation  $\gamma_2(s_b) = 0$ , where in the first case  $J_h^{-1} = 0$ . As expected, in Fig. 2, the MIFTP increases as the node moves away from other co-channel transmitters. It is observed that the proposed MIFTP estimation technique serves as a valid upper bound on allowable interference-free transmit power for secondary transmitters. Improved localization, achieved by using increasing number of SS measurements ( $N$ ), makes the MIFTP estimate tighter. For example, significant gain is seen at shorter distances when  $N = 10$ , compared to  $N = 5$ .

Fig. 3 shows a pictorial illustration of the T-map for a particular channel. In this example, the estimated parameters  $\hat{\theta}_{p_1}$  and  $\hat{\theta}_{p_2}$  of primary transmitters  $p_1$  and  $p_2$  are available, while the true parameters  $\theta_{a_1}$  and  $\theta_{a_2}$  of existing secondary transmitters  $a_1$  and  $a_2$  are also known. All these parameters along with the appropriate CRB estimates are contained in the T-map and are propagated throughout the secondary network via a collaborative network protocol. The circles around  $p_1$ ,  $p_2$ ,  $a_1$  and  $a_2$  represent their respective (estimated or true) coverage regions, inside which potential victim nodes reside. The circle centered at node  $b$  represents a spectrum hole, which is the region where no potential victim node exists. Its radius is the minimum critical distance estimate,  $\hat{D}_b^*$ . Here the critical transmitter is  $t^* = p_1$  and the corresponding critical victim node is  $v^*$ . Since  $\hat{D}_b^* > 0$ , the MIFTP approximation corresponding to the spectrum hole centered at  $b$  is found according using the approach discussed in Section IV. Hence, the information contained in the T-map characterizes the location and size of the available spectrum holes in the spatial domain.

## VI. CONCLUSION

We presented a collaborative OSA scheme whereby multiple primary and secondary transmitters can co-exist in an interference-free condition. The scheme involves distributed sensing of the primary transmitters via the notion of a T-map, a distributed database containing location, power, and error estimates of co-channel nodes. Secondary nodes estimate the power and location of primary transmitters that are located in their vicinity. The effect of co-channel interference in localization is initially ignored and is later taken into account when global information about other co-channel transmitters becomes available via the T-map. The construction, sharing, and updating of the T-map throughout the network is an integral part of our approach, because in addition to quantifying the spectrum holes, the use of collaboration makes the scheme adaptive and robust. We plan to study the parameters that the T-map requires to successfully characterize the spectrum holes in dynamic scenarios, as well as the associated networking issues.

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