Modeling an Opportunistic Spectrum Sharing System with a Correlated Arrival Process

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Abstract— **In an opportunistic spectrum sharing (OSS) wireless network there are two types of users: primary users and secondary users. The primary users own the license for the system bandwidth, while the secondary users opportunistically share the spectrum resources with the primary users. When a secondary user detects a call arrival from a primary user in its current channel, the secondary user leaves the channel immediately and switches to an idle channel, if one is available, to continue the call. Otherwise, the secondary user is preempted. Call arrivals from primary users and secondary users in the OSS system are modeled by a Markovian arrival process (MAP) which captures correlation in the aggregate arrival process consisting of the two types of call arrivals. We derive the stationary probability vector using matrix-analytic methods and obtain expressions for a set of key performance measures. We present numerical results for a sample scenario.**

I. INTRODUCTION

Studies of wireless spectrum usage have shown that large portions of the allocated spectrum are highly underutilized [1]. Frequency agile radios (FARs) are cognitive radios that are capable of detecting idle frequency channels and opportunistically making use of them without causing harmful interference to the authorized or *primary* users [2]. By allowing *secondary* users equipped with FARs to reclaim idle channels, much higher spectrum efficiency can be achieved [3]. More generally, cognitive radios [4] may be capable of opportunistic spectrum access over frequency channels, time slots, or spreading codes. Opportunistic spectrum sharing techniques offer the potential for higher spectrum reuse in commercial, government, and military applications.

Secondary users opportunistically make use of channels that are not occupied by primary users. A secondary user senses when a channel is idle and then makes use of such a channel. If a primary user starts using the channel, the secondary user is preempted. The reliable detection of primary users is a major challenge for the implementation of an OSS system. The spectrum usage of the secondary users is contingent on the requirement that the interference to the primary users be limited to a certain threshold. A number of opportunistic spectrum access (OSA) schemes have been developed recently in the literature [2][5][6][7]. In [2], a framework is developed for modeling the interference caused by FARs employing spectrum access mechanisms based on the Listen-Before-Talk (LBT) scheme. In [5], a multi-channel OFDMA technique is proposed for OSA networks in which the users of the network must contact the OSA Base Station (BS) to gain access to the radio resources. In [6], a measurement-based model is proposed to statistically describe the busy and idle periods of a WLAN. Two different sensing strategies, energy-based detection and feature-based detection, are explored to identify spectrum opportunities. In [7], an admission control algorithm is proposed and performed jointly with power control such that QoS requirements of all admitted secondary users are satisfied while keeping the interference to primary users below the tolerable limit.

In [3], a performance model of an OSS is presented. In this model, secondary users make use of channels that are not used by the primary users. However, primary users have preemptive priority over secondary users. A secondary call that is preempted by a primary call joins a queue and waits for an opportunity to resume its service up to a maximum waiting time. To make the model tractable, call requests from both types of users are assumed to arrive as Poisson processes.

Modern wireless networks (such as 3G, WiMAX, WLAN, UWB, etc.) are targeted to provide integrated services including voice, video, and data. The traffic streams generated by such services may be characterized as being statistically bursty and correlated. The traditional Poisson assumption commonly used to model cellular network traffic does not accurately model bursty, correlated traffic. On the other hand, the Markovian arrival process (MAP) has been found to provide a good representation for bursty and correlated traffic arising in modern wireless networks (cf. [8]-[12]). The MAP encompasses a rich class of point processes as special cases, including the Poisson process, Markov modulated Poisson process (MMPP), PH-renewal process, etc.

This paper[1] focuses on the performance modeling of an OSS system similar to the one in [3] but with a general Markovian arrival process (MAP) to characterize correlation in the traffic arrival processes generated by primary users, secondary users, as well as correlation between the two types of traffic. For tractability, we assume perfect opportunistic spectrum access, i.e., the secondary users are able to move in and out of channels to avoid harmful interference with primary users. We use a multi-dimensional Markov process to model the OSS system and derive the analytical results explicitly using matrix-analytic techniques [9][11].

The remainder of the paper is organized as follows. Section II describes the OSS system and our modeling assumptions. Section III develops a Markovian model to evaluate the

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system performance. Section IV presents a recursive computational algorithm to solve for the stationary state probability vector. Section V derives several performance measures of interest. Section VI presents the numerical results in terms of the obtained performance measures. Finally, Section VII concludes the paper.

II. MODEL DESCRIPTION AND ASSUMPTIONS

Consider a wireless network operating over a given service area. The network owns the license for spectrum usage and hence is referred to as the *primary system*. The users of this network are the primary users. Calls generated by primary users constitute the primary traffic (PT) stream. Next, we introduce another wireless network in the same service area, which opportunistically shares the precious spectrum resource with the existing network. This network is referred to as the *secondary system* and the associated users are called secondary users. Calls generated by secondary users constitute the secondary traffic (ST) stream. A system consisting of the primary and secondary subsystems is called an opportunistic spectrum sharing (OSS) system [3]. The OSS system model introduced here is not restricted to a specific class of wireless networks and can be applied to both infrastructured and infrastureless wireless networks. Without loss of generality, we shall assume that both the primary and secondary systems are infrastructure-based networks with a cellular architecture.

In the OSS system, the spectrum availability for the secondary users depends on the spectrum occupancy of the primary users. A distinct feature of a well-designed OSS system is that the secondary users have the capability to sense channel usage and switch between different channels using appropriate communication mechanisms without causing harmful interference to the primary users. Such functionality could be realized by cognitive radios [4].

Secondary users detect the presence or absence of signals from primary users and maintain records of the channel occupancy status. The detection mechanism may involve collaboration with other secondary users and/or an information exchange with an associated base station (BS) of the secondary system. The PT calls operate as if there are no ST calls in the system. When a PT call arrives to the system, it occupies a free channel if one is available or a channel that is occupied by an ST call; otherwise, it is blocked. Secondary users opportunistically access the channels that are free.

In our proposed model, we assume a perfect signal detection mechanism. When an ST node detects or is informed (by its BS or other ST nodes) of an arrival of a PT call in its current channel, it immediately leaves the channel and switches to an idle channel, if one is available, to continue the call. If at that time all the channels are occupied, the ST call is *preempted* and placed in a queue, which we refer to as the *preemption queue.* The ST call remains in the preemption queue until either the holding time of the call completes or a PT/ST call releases a channel. In the latter case, the ST call at the head of the preemption queue immediately occupies the vacated channel.

The aggregate call arrival process, consisting of PT and ST calls, to the OSS system is modeled by a general MAP, which can capture correlation between interarrival times. The MAP is a generalization of the Poisson process in which the arrivals are governed by an underlying m-state Markov chain. Let g_{ij}^0, $i \neq j$, $1 \leq i, j \leq m$, denote the transition rate from state i to state j without an arrival in the underlying Markov chain. Let g_{ij}^P and g_{ij}^S, $1 \leq i, j \leq m$, denote the transition rate from state i to state j, with a PT call arrival and an ST call arrival, respectively, in the underlying Markov chain. The matrix $G_0 = [g_{ij}^0]$ has nonnegative off-diagonal and negative diagonal elements. The matrices $G_P = [g_{ij}^P]$ and $G_S = [g_{ij}^S]$ consists of nonnegative elements. The matrix $G = G_0 + G_P + G_S$ is the irreducible infinitesimal generator of the m-state Markov chain. The sojourn time in state i is exponentially distributed with parameter λ_i, $1 \leq i \leq m$. At the end of a sojourn in state i, there are three possible transitions [13]:

- A transition to state j without a call arrival occurs with probability q_{ij}^0, $j \neq i$, $1 \leq i, j \leq m$;
- A transition to state j with a PT call arrival occurs with probability q_{ij}^P, $1 \leq i, j \leq m$;
- A transition to state j with an ST call arrival occurs with probability q_{ij}^S, $1 \leq i, j \leq m$.

For each fixed i, the following relation holds:

$$\sum_{\substack{j=1 \\ j \neq i}}^{m} q_{ij}^0 + \sum_{j=1}^{m} q_{ij}^P + \sum_{j=1}^{m} q_{ij}^S = 1.$$

Further, we have $g_{ij}^0 = \lambda_i q_{ij}^0$ for $j \neq i$, $g_{ij}^P = \lambda_i q_{ij}^P$, $g_{ij}^S = \lambda_i q_{ij}^S$ and $g_{ii}^0 = -\lambda_i$, where $1 \leq i, j \leq m$. Note that $(G_0 + G_P + G_S)\mathbf{e} = \mathbf{0}$ holds, where $\mathbf{e}$ is a column vector of m ones.

Let $\boldsymbol{\pi}$ be the stationary state probability vector of the generator G. Then we have $\boldsymbol{\pi} G = 0$ and $\boldsymbol{\pi}\mathbf{e} = 1$. The PT call arrival rate of the MAP is given by $\lambda_P = \boldsymbol{\pi} G_P \mathbf{e}$ and the ST call arrival rate of the MAP is given by $\lambda_S = \boldsymbol{\pi} G_S \mathbf{e}$. When $m = 1$, the MAP reduces to a Poisson process with rate λ_1, composed of two independent Poisson processes with rates $\lambda_1 q_{11}^P$ and $\lambda_1 q_{11}^S$ respectively. When both G_P and G_S are diagonal matrices, the MAP is a Markov Modulated Poisson Process (MMPP), which has been extensively used to describe superposition of data or packetized voice streams [8][14]. The channel holding times or service times of the PT and ST calls are assumed to be exponentially distributed with rate parameters μ_P and μ_S, respectively.

III. PERFORMANCE ANALYSIS

We consider an OSS system in the context of a cellular system wherein events occurring in different cells are assumed to be statistically identical and independent. In our analysis, we consider channel allocation within a single cell.

Suppose there are a total of M channels assigned to the cell.

Let $\{X(t)\colon t \geq 0\}$ be a stochastic process in the considered cell with state space

$$S = \{(0, j) : 1 \leq j \leq m\} \cup \{(i, i_P, j) : 1 \leq i \leq 2M, 0 \leq i_P \leq M, 1 \leq j \leq m\},$$

where $(0, j)$ represents the state with no call in the system and the MAP is in phase j; (i, i_P, j) represents the state with i calls in the system, among which i_P calls are of type PT, and the MAP is in phase j. Here, state i represents the total number of calls (PT or ST) in the system including those in service and ST calls in the *preemption queue*, if any. Since state $(0, j)$ is used to represent an empty system, there is at least *1* call in the system when the system is in state (i, i_P, j), i.e., $i \geq 1$.

When an ST node occupying a channel detects the arrival of a PT call to its current channel and at this time all other channels are occupied, the ST call is preempted and joins the preemption queue. When an ongoing ST or PT call leaves the system, the first ST call in the preemption queue occupies the channel vacated by the completed call. A preempted call remains in the preemption queue until either its holding time expires or it takes over a channel released by a PT or ST call. The maximum number of calls in the preemption queue is M, which corresponds to the case in which M ongoing ST calls have been preempted by the arrivals of PT calls. Thus, we have $1 \leq i \leq 2M$. The number of ST calls occupying channels is given by $i - i_P$ if $i \leq M$ and $M - i_P$, otherwise. In the latter case, the number of ST calls in the preemption queue is given by $i - (M - i_P)$. The number of calls receiving service is given by $\min\{i, M\}$.

Due to the aforementioned assumptions for the arrival processes and service time distributions, the process $\{X(t)\colon t \geq 0\}$ can be characterized as a continuous-time Markov chain (CTMC) with infinitesimal generator given by

$$Q = \begin{bmatrix} E_0 & B_0 & & & \\ D_1 & E_1 & B_1 & & \\ & \ddots & \ddots & \ddots & \\ & & D_{2M-1} & E_{2M-1} & B_{2M-1} \\ & & & D_{2M} & E_{2M} \end{bmatrix}, \tag{1}$$

where E_i ($0 \leq i \leq 2M$) is a matrix representing the absence of transitions from the state in which there are i calls in the system; B_i ($0 \leq i \leq 2M$-1) is a matrix representing the transition rates due to the arrival of a PT or ST call when there are i calls in the system; D_i ($1 \leq i \leq 2M$) denotes a departure of a call when there are i calls in the system. By considering the transitions among the difference states, we can obtain expressions for these matrices as follows.

A. Construction of Matrices B_i

Define $B_0 = [G_P \quad G_S]$. If $1 \leq i \leq M$-1, B_i is an $(i+1) \times (i+2)$ block matrix given by

$$B_i = \begin{bmatrix} B_{i,0}^P & B_{i,0}^S & & & \\ & B_{i,1}^P & B_{i,1}^S & & \\ & & \ddots & \ddots & \\ & & & B_{i,i}^P & B_{i,i}^S \end{bmatrix},$$

where $B_{i,j}^P = G_P$, $0 \leq j \leq i$, represents the state transition rates corresponding to the arrival of a PT call when there are i calls in the system, among which j are PT calls; and $B_{i,j}^S = G_S$, $0 \leq j \leq i$, represents state transition rates corresponding to the arrival of an ST call when there are i calls in the system, among which j are PT calls.

If $M \leq i \leq 2M$-1, B_i is a $(2M-i+1) \times (2M-i)$ block matrix given by

$$B_i = \begin{bmatrix} B_{i,i-M}^P & & & \\ & B_{i,i-M+1}^P & & \\ & & \ddots & \\ & & & B_{i,M-1}^P \\ & & & 0 \end{bmatrix},$$

where $B_{i,j}^P = G_P$, $i\text{-}M \leq j \leq M\text{-}1$, $M \leq i \leq 2M$-1, represents state transition rates corresponding to the arrival of a PT call when there are i calls in the system (including the ST calls in the preemption queue), among which j are PT calls.

B. Construction of Matrices D_i

Define

$$D_1 = \begin{bmatrix} \mu_S I_m \\ \mu_P I_m \end{bmatrix}.$$

If $2 \leq i \leq M$, D_i is an $(i+1) \times i$ block matrix given by

$$D_i = \begin{bmatrix} i\mu_S I_m & 0 & & \\ \mu_P I_m & (i-1)\mu_S I_m & & \\ & \ddots & \ddots & \\ & & (i-1)\mu_P I_m & \mu_S I_m \\ & & & i\mu_P I_m \end{bmatrix},$$

where in each row of D_i, the element $k\mu_P I_m$, $1 \leq k \leq i$, represents state transition rates corresponding to a PT call departure when there are i calls in the system, among which k are PT calls; and the element $(i-k)\mu_S I_m$, $0 \leq k \leq i-1$, represents state transition rates corresponding to the departure of an ST call when there are i calls in the system, among which k are PT calls.

If $M+1 \leq i \leq 2M$, D_i is a $(2M-i+1) \times (2M-i+2)$ block matrix given by

$$D_i = \begin{bmatrix} (i-M)\mu_P I_m & (2M-i)\mu_S I_m & & & & \\ & (i-M+1)\mu_P I_m & (2M-i-1)\mu_S I_m & & & \\ & & \ddots & \ddots & & \\ & & & (M-1)\mu_P I_m & \mu_S I_m & \\ & & & & M\mu_P I_m & 0 \end{bmatrix},$$

where in each row of D_i, the element $k\mu_P I_m$, $i-M \le k \le M$, represents state transition rates corresponding to the departure of a PT call when there are i calls in the system, M calls receiving service, among which k are PT calls; and the element $(M-k)\mu_S I_m$, $i-M \le k \le M$, represents state transition rates corresponding to the departure of an ST call when there are i calls in the system, M calls receiving service, among which k are PT calls.

C. Construction of Matrices E_i

We define $E_0 = G_0$. If $1 \le i \le M$, E_i is an $(i+1) \times (i+1)$ block matrix given by

$$E_i = \begin{bmatrix} E_{i,0} & & & \\ & E_{i,1} & & \\ & & \ddots & \\ & & & E_{i,i} \end{bmatrix},$$

where $E_{i,j}$, $0 \le j \le i$, represents the absence of state transitions when there are i calls in the system, among which j are PT calls, and is given by

$$E_{i,j} = \begin{cases} G_0 - [j\mu_P + (i-j)\mu_S]I_m, \\ \quad if \quad 1 \le i \le M-1,\ 0 \le j \le M-1; \\ G_0 + G_S - [j\mu_P + (i-j)\mu_S]I_m, \\ \quad if \quad i = M,\ 0 \le j \le M-1; \\ G_0 + G_S + G_P - M\mu_P I_m, \\ \quad if \quad i = M,\ j = M. \end{cases}$$

If $M+1 \le i \le 2M$, E_i is a $(2M-i+1) \times (2M-i+1)$ block matrix given by

$$E_i = \begin{bmatrix} E_{i,i-M} & & & \\ & E_{i,i-M+1} & & \\ & & \ddots & \\ & & & E_{i,M} \end{bmatrix},$$

where $E_{i,j}$, $i-M \le j \le M$, represents the absence of state transitions when there are i calls in the system, M calls receiving service, among which at least $(i-M)$ calls are PT calls, and is given by

$$E_{i,j} = \begin{cases} G_0 + G_S - [j\mu_P + (M-j)\mu_S]I_m, \\ if\ M+1 \le i \le 2M-1,\ i-M \le j \le M-1; \\ G_0 + G_S + G_P - M\mu_P I_m, \\ \quad if \quad M+1 \le i \le 2M,\ j = M. \end{cases}$$

IV. COMPUTATION OF STATIONARY STATE PROBABILITY VECTOR

In this section, we derive the stationary state probability vector of the Markov process and obtain several performance measures of interest. Let $p(0,j)$ and $p(i,i_P,j)$ denote the steady-state probability of the system in states $(0,j)$ and (i,i_P,j), respectively. Let $\mathbf{p}_i$ $(0 \le i \le 2M)$ be the steady-state probability vector of the system in equilibrium when there are i calls in the system (including the preempted ST calls), with i_P calls being PT calls, and the arrival process is in phase j. We have $\mathbf{p}_0 = (p(0,1), p(0,2), \cdots, p(0,m))$. For $1 \le i \le 2M$, the components of $\mathbf{p}_i$ are ordered lexicographically. The stationary probability vector is given as follows: $\mathbf{P} = [\mathbf{p}_0, \mathbf{p}_1, \cdots, \mathbf{p}_M, \mathbf{p}_{M+1}, \cdots, \mathbf{p}_{2M}]$, where $\mathbf{p}_i$, $0 \le i \le M$, is a row vector of dimension $(i+1)m$, and $\mathbf{p}_i$, $M+1 \le i \le 2M$, is a row vector of dimension $(2M-i+1)m$. From the equilibrium conditions $\mathbf{P}Q = \mathbf{0}$ and $\mathbf{P}\mathbf{e} = 1$, we have

$$\mathbf{p}_0 E_0 + \mathbf{p}_1 D_1 = 0 \tag{1}$$

$$\mathbf{p}_{i-1} B_{i-1} + \mathbf{p}_i E_i + \mathbf{p}_{i+1} D_{i+1} = 0,\ 1 \le i \le 2M-1, \tag{2}$$

$$\mathbf{p}_{2M-1} B_{2M-1} + \mathbf{p}_{2M} E_{2M} = 0. \tag{3}$$

Solving the above equations, we obtain the recurrence formula

$$\mathbf{p}_i = \mathbf{p}_{i+1} D_{i+1} C_i^{-1},\quad 0 \le i \le 2M-1, \tag{4}$$

$$\mathbf{p}_{2M} C_{2M} = 0, \tag{5}$$

where $C_0 = -E_0$ and

$$C_i = -E_i - D_i C_{i-1}^{-1} B_{i-1},\ 1 \le i \le 2M. \tag{6}$$

We point out that the recursive solution approaches used in [13][15] are not applicable here since the block-matrix structure is different. Following the above recursive equations, the stationary probability vector $\mathbf{P}$ can be numerically determined. We summarize the procedure for computing the stationary state probability vector as follows:

Step 1: Compute the matrices C_i, $0 \le i \le 2M$, by using (6).

Step 2: Determine the probability vector $\mathbf{p}_{2M}$ by using (5).

Step 3: Compute the remaining vectors $\mathbf{p}_i$, $0 \le i \le 2M-1$, by using (4).

Step 4: Normalize the probability vectors $\mathbf{p}_i$, $0 \le i \le 2M$. Then obtain the state probability vector $\mathbf{P}^*$ by normalizing $\mathbf{P} = [\mathbf{p}_i : 0 \le i \le 2M]$ as follows: $\mathbf{P}^* = \dfrac{\mathbf{P}}{\mathbf{P}\mathbf{e}}$.

V. PERFORMANCE MEASURES

Given the stationary state probability vector, we can determine various performance measures of interest.

A. Blocking Probability of ST Calls

The ST call blocking probability, denoted by B_S, is defined as the probability that all M channels are occupied by either PT or ST calls and a new ST call request arriving to the system must be blocked. We have

$$B_S = \sum_{i=M}^{2M}\sum_{i_P=0}^{M}\sum_{j=1}^{m} p(i,i_P,j) = \sum_{i=M}^{2M}\mathbf{p}_i\mathbf{e}\,. \tag{7}$$

B. Blocking Probability of PT Calls

The PT call blocking probability, denoted by B_P, is defined as the probability that all M channels are occupied by PT calls and a new PT call request arriving to the system must be blocked. We have

$$B_P = \sum_{i=M}^{2M}\sum_{j=1}^{m} p(i,M,j) = \sum_{i=M}^{2M}\mathbf{p}_i\Big|_{i_P=M}\mathbf{e}. \tag{8}$$

C. Total Channel Utilization

The total channel utilization, denoted by η, is defined as the ratio of the mean number of occupied channels to the total number of channels. We find that

$$\eta = \frac{1}{M}\left\{\sum_{i=1}^{M} i\,\mathbf{p}_i\mathbf{e} + \sum_{i=M+1}^{2M} M\,\mathbf{p}_i\mathbf{e}\right\} = \frac{1}{M}\sum_{i=1}^{2M}\min\{i,M\}\,\mathbf{p}_i\mathbf{e}. \tag{9}$$

D. Mean Number of ST Calls in the Preemption Queue

It is of interest to compute the mean number, N_V ,of ST calls in the preemption queue in steady-state. This metric can be used to evaluate the performance of the secondary system and to determine an appropriate range of parameter values for the arrival and departure processes. We have

$$N_V = \sum_{i=M+1}^{2M}(i-M)\,\mathbf{p}_i\mathbf{e}. \tag{10}$$

E. Mean Dropping Ratio of the Ongoing ST Calls

As mentioned earlier, when an ST node detects an arrival of PT call in its current channel and at that time all other channels are occupied, the ST call will be placed in the preemption queue. We define the mean dropping ratio, γ, of the ongoing ST calls in steady-state, as the ratio of the mean number of ST calls in the preemption queue N_V to the total number of ST calls in the system (including those in the preemption queue) N_{total}, in equilibrium, i.e.,

$$\gamma = \frac{N_V}{N_{total}}\quad , \tag{11}$$

where $$N_{total} = \sum_{i=1}^{2M}(i-i_P)\,\mathbf{p}_i\mathbf{e}. \tag{12}$$

VI. NUMERICAL RESULTS

We present some numerical results with respect to the performance measures obtained in Section V under the following parameter settings: $M = 12$, $\mu_P = 5$, $\mu_S = 8$. The MAP parameters are set as follows: $m = 3$,

$$G_P = \begin{bmatrix} 0.1\lambda_1 & 0.2\lambda_1 & 0.1\lambda_1 \\ 0.1\lambda_2 & 0.15\lambda_2 & 0.1\lambda_2 \\ 0.15\lambda_3 & 0.1\lambda_3 & 0.1\lambda_3 \end{bmatrix},\ G_S = \begin{bmatrix} 0.1\lambda_1 & 0.2\lambda_1 & 0.2\lambda_1 \\ 0.15\lambda_2 & 0.2\lambda_2 & 0.2\lambda_2 \\ 0.2\lambda_3 & 0.2\lambda_3 & 0.15\lambda_3 \end{bmatrix},$$

$$G_0 = \begin{bmatrix} -\lambda_1 & 0.05\lambda_1 & 0.05\lambda_1 \\ 0.05\lambda_2 & -\lambda_2 & 0.05\lambda_2 \\ 0.05\lambda_3 & 0.05\lambda_3 & -\lambda_3 \end{bmatrix}.$$

We set $\lambda_1 = \lambda_2 = \lambda_3 = \lambda$. Fig. 1 shows the impact of the call arrival rates λ_P and λ_S (through the MAP parameter λ) on the PT and ST call blocking probabilities B_p and B_s. As expected, as the MAP parameter λ is increased, λ_P and λ_S increase linearly, leading to an increase in both blocking probabilities B_p and B_s. For high call arrival rates, the performance of the secondary system deteriorates due to the lack of available channels.

Fig. 2 shows how the channel utilization η changes as a function of the call arrival rate (through the MAP parameter λ) with different mean channel holding times specified by μ_p and μ_s. As call arrival rate increases, so too does the channel utilization η. As the mean channel holding time for either type of call increases, so too does the channel utilization η, which agrees with intuition.

Fig. 3 shows the mean dropping ratio of the ongoing ST calls γ as a function of the call arrival rate with different values of μ_p and μ_s. As the call arrival rate increases, the mean dropping ratio γ increases. As more PT calls enter the system, fewer channels are available for ST calls. Thus, a preempted ST call has a smaller chance of obtaining an idle channel to continue its call. When the holding time of a PT call, $1/\mu_P$, is decreased with $1/\mu_S$ kept fixed, the mean dropping ratio γ decreases because the preempted ST calls will have a better chance of obtaining an idle channel. At the same time, the mean number of ST calls in equilibrium, N_{total} , will increase due to the higher availability of channels for ST calls. However, when the holding time of a ST call $1/\mu_S$ is decreased (with $1/\mu_P$ kept fixed), the impact on the ratio γ is not obvious since both N_v and N_{total} will be decreased.

VII. CONCLUSION

We have studied an opportunistic spectrum sharing (OSS) wireless system consisting of two types of users: primary users and secondary users. The secondary users opportunistically share a set of spectrum resources with the primary users over a coverage area. When a secondary user detects the arrival of a primary user's call in its current channel, it will leave the channel immediately and switch to an idle channel, if one is available, to continue the call. Otherwise, it will be placed in a preemption queue. Call arrivals from primary users and secondary users in the OSS system are modeled by a general Markovian arrival process (MAP) which can characterize correlation in the arrival process. We derived the stationary state probability vector using matrix-analytic techniques and obtained several key performance measures. The analytical results derived in this paper can be used to dimension OSS systems carrying traffic with a wide range of characteristics.

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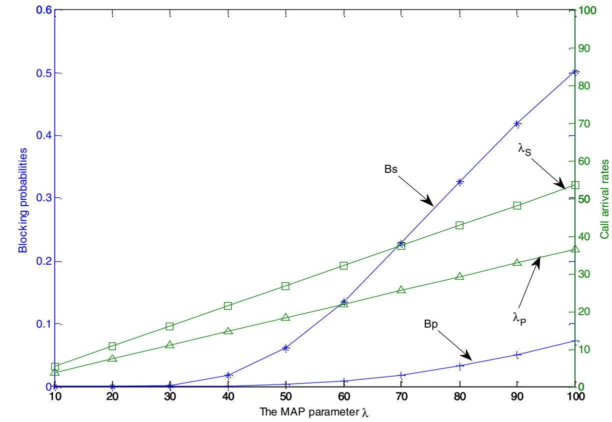

Fig. 1. PT and ST call blocking probabilities vs. call arrival rates.

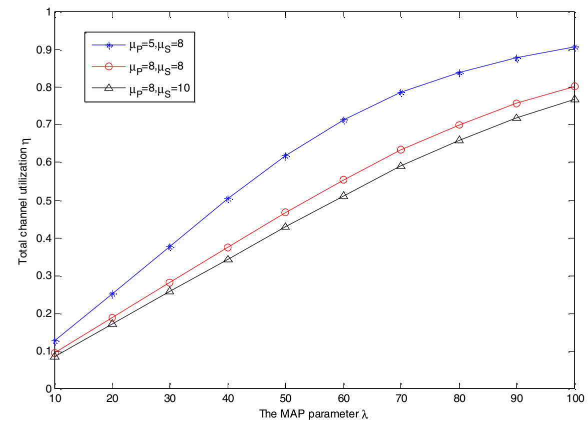

Fig. 2. Total channel utilization vs. call arrival rate (via parameter λ).

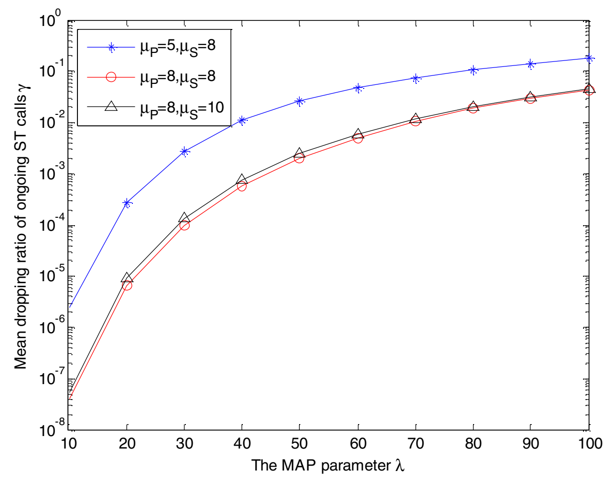

Fig. 3. Mean dropping ratio γ vs. call arrival rate (via parameter λ).