Online Appendix to: Reduction of Closed Queueing Networks for Efficient Simulation

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This document is a supplement for the article by Shortle et al. [2009]. The supplement gives a small numerical example to illustrate the network reductions given in the main article. The purpose is to aid the reproducibility of the results given there.

This supplement gives a small numerical example to illustrate the network reductions given in Shortle et al. [2009]. The purpose is to aid the reproducibility of the results given in the main article.

Table I shows parameters for a 12-node network. The routing probabilities were obtained from flight data over a one-month period at 12 airports. Specifically, $p_{i,j} \propto n_{i,j}$, where $n_{i,j}$ is the observed count of flights from airport i to airport j. This network is intended for illustration, not to actually model the air transportation system. The first four nodes represent major airports, while the remaining nodes represent moderately sized airports. A natural choice for $\mathcal{C}$ is $\{1, 2, 3, 4\}$. The remaining network parameters are chosen somewhat arbitrarily: the number of customers (airplanes) in the system is $M = 120$; the service parameters μ_i and c_{si}^2, are given in the table. Table II shows the approximate queueing metrics obtained using Algorithm A. Table III shows the results of Reduction 1 applied to the sample network. Table IV shows the results of Reduction 2 to the sample network. The table only shows the transition probabilities from $\mathcal{C}$ to $\mathcal{R}^*$.

Table I. Example Network: Routing Probabilities $p_{i,j}$ and Service Parameters

		1	2	3	4	5	6	7	8	9	10	11	12	μ_i	c_{si}^2
$\mathcal{C}$	1	—	.17	.17	.05	.02	.07	.05	.09	.15	.09	.12	.02	46	.5
	2	.14	—	.18	.14	.01	.11	.10	.08	.08	.03	.06	.07	55	.5
	3	.23	.30	—	.11	—	.05	.05	.03	.09	.03	.08	.03	31	.5
	4	.15	.44	.25	—	—	.06	—	.02	.06	.02	—	—	17	.5
$\mathcal{R}$	5	.32	.21	.01	.02	—	—	.02	.32	.10	—	—	—	22	.5
	6	.19	.33	.10	.05	—	—	.10	.06	.10	.02	.05	—	22	.5
	7	.16	.34	.09	—	—	.12	—	.04	.14	.03	.06	.02	22	.5
	8	.26	.25	.07	.02	.06	.07	.05	—	.11	.07	.02	.02	22	.5
	9	.26	.19	.12	.03	.01	.08	.10	.07	—	.07	.06	.01	22	.5
	10	.41	.13	.10	.03	—	.03	.04	.09	.14	—	.03	—	22	.5
	11	.34	.20	.19	—	—	.06	.06	.02	.09	.02	—	.02	22	.5
	12	.15	.54	.14	—	—	—	.05	.04	.03	—	.05	—	22	.5

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Table II. Approximate Intermediate Variables for Example Network Using Algorithm A

Node	1	2	3	4	5	6	7	8	9	10	11	12
λ_i	41.15	48.24	30.34	14.24	2.37	16.18	14.51	14.02	21.89	9.90	14.14	6.18
ρ_i	0.89	0.88	0.98	0.84	0.11	0.74	0.66	0.64	0.99	0.45	0.64	0.28
c_{ai}^2	0.93	0.91	0.94	0.96	0.99	0.97	0.97	0.97	0.96	0.98	0.97	0.98
c_{di}^2	0.59	0.59	0.52	0.64	0.99	0.71	0.76	0.78	0.50	0.88	0.77	0.94
$p_{i,\mathcal{R}}$	0.61	0.54	0.36	0.16								
$p_{\mathcal{R},i}$	0.39	0.40	0.17	0.03								
$c_{a\mathcal{R},i}^2$	0.93	0.93	0.97	0.99								

Table III. Reduction 1 Applied to Example Network

$\hat{p}_{i,j}$	1	2	3	4	$\mathcal{R}^*$	μ_i
1	—	0.17	0.17	0.05	0.61	46
2	0.14	—	0.18	0.14	0.54	55
3	0.23	0.30	—	0.11	0.36	31
4	0.15	0.44	0.25	—	0.16	17
$\mathcal{R}^*$	0.39	0.40	0.17	0.03	—	64.598

Table IV. Reduction 2 Applied to Example Network

$\hat{p}_{i,j}$	1*	2*	3*	4*
1	0.240	0.245	0.105	0.020
2	0.212	0.217	0.093	0.018
3	0.141	0.145	0.062	0.012
4	0.063	0.064	0.028	0.005
μ_{i^*}	26.507	27.139	11.635	2.192
$c_{si^*}^2$	0.930	0.933	0.966	0.989

REFERENCES

SHORTLE, J. F., MARK, B. L., AND GROSS, D. 2009. Reduction of closed queueing networks for efficient simulation. *Trans. Model. Comput. Simul. 19*, 3.