Spectrum Sensing Using Markovian Models

2

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Contents

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Abstract

Markovian models, as well as other statistical models, have been applied in the context of cognitive radio communications to characterize user activity in a given spectrum band and to develop algorithms for temporal spectrum sensing. In this chapter, we discuss spectrum sensing based on Markovian models. We provide an overview of the related literature and then discuss the application of discrete-time Markov chain models to spectrum sensing, in particular the hidden bivariate Markov chain. We focus on the modeling of cognitive radio channels using Markov chains, spectrum detection, and parameter estimation. We then discuss various spectrum sensing scenarios in which the Markovian models are used. Finally, we discuss open problems and topics for further research related to spectrum sensing using Markovian models.

Introduction

The conventional approach to spectrum management is to partition the spectrum into bands and issue licenses for spectrum usage in those bands. Studies of spectrum usage have shown that this spectrum allocation paradigm often results in severe underutilization of the spectrum; see, e.g., [22, 45]. In opportunistic or dynamic spectrum access, a band of licensed spectrum can be used by unlicensed users whenever it is not being used by licensed users. The licensed user is often referred to as the primary user, whereas the unlicensed user is called the secondary user. To take advantage of the portions of the spectrum unused by the primary users, also known as spectrum holes, the secondary users must be capable of dynamically switching their transmissions among different frequency bands, i.e., they must be frequency agile. In addition, the secondary users must be capable of sensing the radio environment to determine the spectrum hole opportunities for dynamic spectrum access. Such capabilities are realized in cognitive radio technologies [26]. A group of communicating secondary users equipped with cognitive radios forms a cognitive radio network. In the context of dynamic spectrum access, the goal of the cognitive radio network is to maximize spectrum utilization while avoiding harmful interference to the primary users.

Dynamic spectrum access can be seen as a “fix” to the problem over spectrum underutilization caused by static allocation of spectrum to licensed users. The primary users maintain strict priority over the secondary users with respect to access to the licensed spectrum. The onus is on the secondary users to identify spectrum holes and transmit in such a way as to avoid harmful interference to the primary users. In a more general dynamic spectrum *sharing* framework, a given spectrum band may be shared among different groups of users rather than being licensed to a certain group of primary users. The spectrum may be shared according to some criterion of fairness, and the different groups of users may collaborate with each other to maximize overall spectrum efficiency while achieving their own individual communication goals. As in dynamic spectrum access, the users in a dynamic spectrum sharing network must be capable of sensing the radio environment to determine spectrum holes and be frequency agile.

In this chapter, we focus on the problem of spectrum sensing, specifically the temporal aspect of identifying when spectrum holes occur in time. Our treatment of temporal spectrum sensing is based on Markovian models of user activity. To simplify the discussion, we will use the terminology of primary users and secondary users associated with the dynamic spectrum access paradigm, but the same principles of spectrum sensing will be applicable in a more general dynamic spectrum sharing setting. We will mainly look at applying Markovian models to characterize spectrum usage, detection of spectrum activity, and parameter estimation. For a given user, there is an important trade-off between devoting resources toward spectrum sensing vs. actually using spectrum for transmission. In the terminology of multiarmed bandit problems, this is the trade-off between "exploration" and "exploitation." To limit the scope of this chapter, we shall focus almost exclusively on the exploration aspect and consider only discrete-time models in detail. The spectrum sensing techniques discussed here could be applied in a dedicated spectrum monitoring infrastructure independently from the users. In this setting, the users would consult the spectrum monitoring system to determine spectrum availability, rather than perform the spectrum sensing themselves. Nevertheless, we will touch upon some of the related work in the literature that deals with both the exploration and exploitation aspects of spectrum sensing.

The remainder of this book chapter is organized as follows. In section "Background and Overview of Related Work," we provide an overview of the literature related to spectrum sensing using Markovian models. In section "Markovian Models," we introduce some terminology and notation for Markovian models as they are applied to temporal spectrum sensing. We focus on the hidden bivariate Markov chain and discuss the associated problems of spectrum detection and parameter estimation. In section "Spectrum Sensing Scenarios," we discuss various spectrum sensing scenarios in which the Markovian models can be applied. In section "Research Challenges and Open Problems," we consider some future challenges and areas of further research involving spectrum sensing via Markovian models. A brief summary and concluding remarks are given in section "Conclusion."

Background and Overview of Related Work

In this section, we provide background on applications of Markovian models to spectrum sensing and an overview of the related work in the literature. Some of the material discussed at a high level in this section is treated in greater detail in subsequent sections.

Applications of Markovian Models

Markovian models have been applied in many different areas, including network traffic modeling [23], speech processing [17], and ion channel current modeling [4], just to name a few. In such applications, some aspect of the system or signal of interest is represented by a Markov process either in discrete time or continuous time. Depending on the application, a discrete-time or a continuous-time model may be preferred for various reasons such as model fidelity, mathematical tractability, or computational complexity. Typically, the number of states of the Markov process is assumed to be finite. In general, Markovian models are attractive because of their mathematical tractability relative to other statistical models and their wide applicability. The fidelity of the Markovian model representation can sometimes be improved by increasing the number of states of the Markov process, but often at the expense of higher computational complexity and the need for a larger training set for parameter estimation.

In some applications, the purpose of the Markovian model is to represent the system or signal as closely as possible in a statistical sense. In other applications, the Markovian model may be only a means to an end, and accuracy of the model itself is of less importance than that of the final result, though the two are obviously closely related. In queueing applications, for instance, the arrival process may not need to be modeled with high accuracy if only the mean queue length is desired. For such a situation, a Poisson arrival process may be sufficient. If the queue length distribution is desired, a more sophisticated model such as the Markov-modulated Poisson process (MMPP) may be necessary. In teletraffic applications, the Poisson process has been found to be sufficiently accurate for modeling call arrivals, but does not provide a good model for bursty packet traffic. The MMPP model can represent the burstiness in packet arrivals [27], but may not be sufficiently rich to represent other phenomena such as long-range dependence or self-similarity, which have been observed in empirical studies of packetized traffic. Modeling of such phenomena takes us outside the realm of Markovian models, so some authors have devised approaches to approximate, in a local sense, characteristics such as long-range dependence using Markovian models.

Markovian Models in Cognitive Radio Systems

In a cognitive radio system, where the secondary user must vacate a channel before a primary user accesses it, statistical prediction of the future primary user state can reduce the probability of interference. Therefore, a Markovian model of the primary user can be used for state prediction, which in turn can prevent unintended collisions. Several cognitive radio media access control methods have been developed which model the primary user as a Markovian process. In [58], the primary user is modeled as a discrete-time partially observable Markov decision process (POMDP), where only part of the state of the system is observed. The system consists of a set of independent channels, each occupied by a primary user that is modeled as a two-state discrete-time Markov chain. At a given time slot, the states of only a small subset of the total set of channels can be observed. Based on the partially observed state, the secondary user makes a decision on which channel or channels to sense or access in the next time slot. The Markov chain parameters for the primary users on the channels are assumed to be known. The basic POMDP model is augmented to account for errors in observing the partial system state.

Markovian models have been used to model primary users in optimization of exploration/exploitation of channels in cognitive radio systems. In [39, 59], a restless multiarmed bandit model is used to model dynamic spectrum access in the presence of multiple primary users on different channels. Solutions are presented to determine how much time should be spent sensing each channel with the objective of maximizing system throughput. In these works, the primary users are modeled by two-state discrete-time Markov chains, but estimation of the Markov chain parameters is not performed explicitly. In [39], estimation of the stationary distribution in the formulation of the reward function is performed.

Hidden Markov Models

In cognitive radio applications, the state of the primary user must be inferred by observing the signal through a noisy channel. When the primary user is modeled as a Markov chain, and the process is observed through a memoryless channel, the resulting model is a hidden Markov process. The bivariate process which comprises the Markovian primary user chain and the observations from the channel observations is a Markov process. The observable process alone is not Markov. The hidden Markov process is a natural model, which allows for inference of the underlying state, while still allowing for prediction of future states. A hidden Markov model consists of the underlying or hidden state process, together with an observable process that is conditionally dependent on the underlying state. The joint bivariate process retains the Markov property, so HMMs fall under the purview of Markovian models. As an additional benefit, HMMs allow for smoothing of decisions by performing maximum likelihood sequence estimation with the Viterbi algorithm, which can reduce the probability of detection error [20]. The HMM has been applied in many different fields including speech and image processing. A comprehensive review of HMMs is given in [20].

HMMs were first introduced as a model for spectrum sensing in cognitive radio networks independently in [1] and [40]. Although an HMM may use any conditional distribution to model signal impairments, the conditional normal distribution is of particular interest for its ability to model additive white Gaussian noise, Rayleigh fading [7], and lognormal shadowing [38].

Parameter Estimation for Markovian Models

Much of the research regarding cognitive radio media access using Markovian models relies on knowledge of the Markov chain parameters, either the state transition rates for continuous-time Markov chains or the state transition probabilities for discrete-time Markov chains. In practice, these parameters are unlikely to be known a priori, so a secondary user would have to perform parameter estimation as part of its spectrum sensing process. In [50], multichannel Markov parameter estimation for cognitive radio is considered. Multiple channels, each containing a single primary user, are observed by the secondary user. The primary user is modeled as a continuous-time Markov chain, and each channel is sensed sequentially using a maximum likelihood estimator [2]. Per-channel sensing times are allocated such that the total variance across all channels is minimized.

For HMMs, additional parameters of the conditional distributions must be estimated with the Markov chain parameter. The Baum algorithm (Also known as the Baum-Welch algorithm.), described in detail in [20], is used to estimate the Markov chain transition matrix and the conditional distribution. The Baum algorithm is a specific realization of the expectation-maximization algorithm [15]. An initial parameter is specified, and the algorithm alternates between computing the state probability distribution for each data point and reestimating the parameters.

The Baum algorithm is considered an *offline* parameter estimation algorithm because it requires a relatively large record of sample data. Reestimation of parameters if new data is received would require iterating again over the entire data set. In [44] and [46], recursive HMM parameter estimators were proposed. Recursive parameter estimation allows for *online* processing of data, where single samples or small blocks of samples may be used to update the HMM parameter estimates, rather than a large block of sample data.

Multivariate Markovian Models

As discussed above, the HMM is an example of a bivariate Markovian model. The underlying Markov chain of an HMM has geometrically distributed so journ times in each state. In many applications, including spectrum sensing, the state sojourn times may have non-geometric distributions such that the HMM is not a suitable model. A more general model that has been proposed is the hidden semi-Markov model [55], in which the sojourn time distributions are explicitly represented in the parameter of the model. The hidden semi-Markov model, however, is not Markovian. Consequently, parameter estimation for the hidden semi-Markov model has significantly higher computational complexity.

As an alternative to the semi-Markov model, a bivariate Markov chain can be used to model the underlying state. The bivariate Markov chain consists of two processes: one represents the observed state and the other represents phases during which the first process remains in a given state. This gives rise to discrete phase-type state sojourn time distributions. By observing a bivariate Markov chain through a channel, we obtain a hidden bivariate Markov chain, which generalizes the HMM, but remains within the realm of Markovian models, i.e., the hidden bivariate Markov chain is a trivariate Markov process. A review of bivariate and multivariate Markov processes is given in [18].

The hidden bivariate Markov chain enjoys many of the same properties of an HMM. For example, the Baum algorithm can be used to perform parameter estimation for the hidden bivariate Markov chain. The hidden bivariate Markov chain may be considered a special case of a hidden semi-Markov model, in which the sojourn time distributions are of discrete-phase type. Unlike the hidden semi-Markov model, however, the sojourn time distribution is implicit in the model, rather than specified as a separate component of the model parameter. With a single underlying state, the bivariate Markov chain is equivalent to a standard univariate Markov chain. Increasing the number of phases or order of the bivariate Markov chain allows for modeling the state sojourn time distributions with more general phase-type distributions. On the other hand, the parameter estimation becomes more computationally complex, and more observation data is required to avoid overfitting the model.

In [38], hidden bivariate Markov chains were applied to spectrum sensing, and it was shown that when predicting many steps in the future, the bivariate Markov process was substantially more accurate. In [48], the work in [38] was extended using a recursive parameter estimation algorithm to perform online spectrum sensing. The algorithm from [44] was extended to perform parameter estimation of hidden bivariate Markov processes.

Markovian Models

Markov Chains

A Markov chain is a discrete-time random process with finite or countably infinite alphabet. Each letter of the alphabet is commonly referred to as a *state*, and the collection of states, i.e., the alphabet, is commonly referred to as the *state space*. The Markov chain may start from one of the states and subsequently visits any state that is reachable from the present state. A state is reachable if the probability of jumping to that state in one or more steps is positive. Self-transitions are allowed. In a Markov chain, future states are conditionally independent of past states given the current state. Thus, a Markov chain is characterized by the initial distribution which represents the probability of the chain to start from any given state and by the transition matrix which contains all conditional probabilities of the chain to visit any state given any current state. Each such conditional probability is referred to as the *transition probability*.

A discrete-time memoryless random process with finite alphabet is a particular Markov chain. Markov chains enable dependence of the process at various times and thus are more suitable for many applications. Furthermore, since their mathematical structure is relatively simple, they are amenable to mathematical analysis, and they are well understood.

We denote the Markov chain by $X = \{X_0, X_1, \ldots, X_t, \ldots\}$ where $X_t \in \mathbb{X}$ and $\mathbb{X} = \{0, 1, \ldots, d\}$ denotes the state space for some finite integer d. In cognitive radio applications, the process X represents the status of the primary user at any given moment. Thus, $X_t = 0$ when the primary user is idle at time t and $X_t = 1$ when the primary user is active. Clearly, in that case $d = 1$. It is possible to choose $d > 1$ to refine the description of the status of the primary user. For example, various states may represent different levels of transmission power by the primary user. We assume that the Markov chain is homogeneous and irreducible. Homogeneity means that transition probabilities are independent of time. Irreducibility means that each state may be reached from any other state, that is, $\mathbf{P}(X_{t+k} = j | X_t = i) > 0$ for any $i, j \in \mathbb{X}$, any $t \geq 0$, and some $k > 0$. We use the row vector $\pi = \text{row}(\pi_0, \pi_1, \ldots, \pi_d)$ to denote the initial distribution of the process X and $A = \{a_{ij}, i, j = 0, 1, \ldots, d\}$ to denote the *transition matrix* of X. For given (i, j), π_i is the probability of $X_0 = i$, and a_{ij} represents the conditional probability of $X_t = j$ given $X_{t-1} = i$ for any $t \geq 1$. From the Chapman-Kolmogorov theorem, A^k represents the k-step transition matrix. That is, $A^k(i, j) = \mathbf{P}(X_{t+k} = j \mid X_t = i)$ for any positive integer k and any $t \geq 0$. The Markov chain is a stationary process if and only if its initial distribution π satisfies $\pi = \pi A$. In that case, π is called the stationary distribution of X, and $\pi_i = \mathbf{P}(X_t = i)$ for any $t \geq 0$. By the Perron-Frobenius theorem [29, p. 536], a sufficient condition for a finite state Markov chain to have a unique stationary distribution is that the chain be irreducible. A well-known result for an irreducible aperiodic finite state Markov chain is

$$\lim_{k \to \infty} A^k = \Pi \tag{1}$$

where Π is a matrix with identical rows each equal to the stationary distribution π. A Markov chain is *aperiodic* if the greatest common divisor of its returning epochs to a given state is one. This result shows that for sufficiently large t, $\mathbf{P}(X_t = j \mid X_{t-1} = i) = \pi_j$, $j = 1, \ldots, d$, regardless of i, where π is the stationary distribution of the chain.

There are many excellent books on Markov chains. The book by [5] contains an elementary chapter on finite-state Markov chains. The book by Kemeny and Snell [28] is very accessible, and there is the classic and more advanced book by Çinlar [11].

Hidden Markov Models

Normally, the process X cannot be observed directly. When the channel is sensed, noise is inevitably present at the cognitive radio receiver. A Markov chain observed through noisy memoryless channel is commonly referred to as a hidden Markov model (HMM). An HMM is not a Markov chain, and its analysis is far more complex. HMMs, however, have numerous applications, and their statistical theory is very well understood. A review of HMMs may be found in [20]. Let $Y = \{Y_0, Y_1, \ldots, Y_t, \ldots\}$ denote the received signal by the cognitive radio receiver. The HMM is characterized by (π, A) as well as by the transition density of the channel. This is the conditional density of Y_t given X_t. We denote this conditional density by $b(y_t \mid x_t)$. The density $b(y_t \mid x_t)$ could be normal, exponential, Poisson, Gamma, etc. Let $B = \{b(y_t \mid x_t), x_t = 0, 1, \ldots, d\}$ denote the collection of all possible densities associated with the various states. In cognitive radio applications, when $x_t = 0$, $b(y_t \mid x_t)$ represents the density of the received signal when the primary user is idle, and when $x_t = 1$, $b(y_t \mid x_t)$ represents the density of the received signal when the primary user is active. Motivated by a central limit theorem, the density $b(y_t \mid x_t)$ is usually assumed normal with mean and variance dependent on the value of x_t. If we denote the mean and variance by μ_{x_t} and $\sigma^2_{x_t}$, respectively, then the parameter of the HMM is given by $\phi = \big(\pi, A, \{(\mu_i, \sigma_i^2), i = 0, \ldots, d\}\big)$. Hidden Markov models in the forms of a Markov chain observed through a channel with memory are also possible.

Let y^n denote a realization of the observation sequence $Y^n = \{Y_0, Y_1, \ldots, Y_n\}$ at the input of the cognitive radio receiver. The density of y^n is then given by

$$p(y^n; \phi) = \sum_{x_0,\ldots,x_n} \pi_{x_0} \prod_{t=1}^{n} a_{x_{t-1}x_t} b(y_t \mid x_t) \tag{2}$$

The parameter ϕ of the HMM may be estimated in an unsupervised offline manner from some training data. This is usually done by the Baum algorithm which is the earliest form of the expectation-maximization (EM) algorithm. The Baum algorithm is an iterative approach for generating a sequence of parameter estimates with increasing likelihood values unless a fixed point in the parameter space is reached. In the latter case, the algorithm is terminated, and the fixed point is a stationary point of the likelihood function. The algorithm is not guaranteed to reach a fixed point and hence is practically terminated when the relative likelihood values in two consecutive iterations fall below a preset threshold. Conditions for convergence of the sequence of estimates generated by the EM algorithm were given by Wu [54].

Given a value ϕ_κ of the true parameter of the HMM at the conclusion of the κth iteration, a new estimate is obtained as

$$\begin{aligned}\phi_{\kappa+1} &= \arg\max_\phi E\{\log p(x^n, y^n; \phi) \mid y^n; \phi_\kappa\} \\ &= \arg\max_\phi \sum_{x^n} p(x^n \mid y^n; \phi_\kappa) \log p(x^n, y^n; \phi).\end{aligned} \tag{3}$$

The density $p(x^n, y^n; \phi)$ is given by the summand of (2). Implementation of the Baum algorithm requires the density $p(x_t \mid y^n; \phi_\kappa)$, $t = 1, 2, \ldots, n$, which is efficiently calculated using the so-called forward-backward algorithm. The forward density is given by $p(x_t, y^t; \phi_\kappa)$, $t = 0, 1, \ldots, n$, and the backward density is given by $p(y_{t+1}^n \mid x_t; \phi_\kappa)$, $t = n, n-1, \ldots, 0$, where $p(y_{n+1}^n) = 1$ and $y_{t+1}^n = \{y_{t+1}, \ldots, y_n\}$. Both densities are calculated recursively in forward and backward modes, respectively. Progressive scaling is required for better numerical stability. Scaling of the forward density, for example, results in recursive evaluation of $p(x_t \mid y^t; \phi_\kappa)$. We demonstrate the recursive calculation of the forward-backward formulas and the scaling procedure in section "Forward-Backward Matrix Recursions." The Baum algorithm is well known for its slow convergence which cannot be controlled by a choice of a step size as in Newton's methods for maximizing a function.

In cognitive radio applications, a fading channel with thermal additive noise is assumed. The measurements $\{Y_t\}$ constitute the logarithm of the power of the signal in a given narrowband portion of the available spectrum. When the state of the primary user is $X_t = a$, then Y_t is assumed normal with mean μ_a and variance σ_a^2. The samples $\{Y_t\}$ are assumed statistically independent. This model is motivated by a central limit theorem developed in [21]. The relation between $\{\mu_a, \sigma_a^2\}$ and the parameters of the fading additive noise channel is nontrivial. In [21, Corollary 5.6.3], the statistics of the logarithm of the smoothed periodogram of a stationary process with small dependence span were studied. The power of each narrowband signal may be seen as a value of the smoothed periodogram of a broadband process measured at a particular frequency. It was shown in [21] that the log-smoothed periodogram at a given frequency is asymptotically normal with mean that depends on the underlying power spectral density and a constant variance that is independent of the underlying power spectral density. In the proposed model, we allow both the mean and variance of each Y_t to depend on the state of the primary user, and hence on the underlying power of the received signal, in order to accommodate possible deviations from the model of [21]. If the variance of Y_t is somewhat independent of the underlying hypothesis, then that should be reflected in its estimated values.

When the active/idle process of the primary user in cognitive radio is represented by a Markov chain as described in this article, the sojourn time of the primary user in each of the two states has a geometric distribution with parameter that depends on the present state. The geometric distribution presents an unrealistic restriction since it does not conform with the typical sojourn time distributions of the primary user. That well-known fact in cognitive radio applications, as well as in other applications such as speech recognition, has led to the use of semi-Markov models [10,11,55]. In these models, which are not Markov processes, a desired sojourn time distribution is imposed. A semi-Markov process may be seen as a pair of processes comprising a Markov jump process for the states and a sequence of conditionally independent sojourn times given the sequence of states. Estimation of the parameter of a semi-Markov process is far more complicated than that of an HMM. This difficulty may be circumvented by substituting the Markov chain of the HMM by a bivariate Markov chain.

Bivariate Markov Chains

A bivariate Markov chain is a pair of discrete-time finite state random processes that are jointly Markov. Each of the individual processes is not necessarily Markov. When a bivariate Markov chain is used in cognitive radio sensing, one of the two processes represents the state process of the primary user while the other is an auxiliary process which endows the primary process with some desired statistical properties. A particularly useful property is a new distribution of the sojourn time of the primary process in each of its state. This distribution is phase type and is far more general than the geometric sojourn time distribution of the univariate Markov chain. The set of discrete phase-type distributions is dense in the set of distributions on $0, 1, 2, \ldots$. This means that every distribution in that family is either a phase type or it can be approximated arbitrarily well by a phase-type distribution. The family of phase-type distributions includes mixtures of convolutions of geometric distributions. Intuitively, either a jump of the two processes comprising the bivariate Markov chain or a joint jump of the two processes constitutes a jump of the bivariate Markov chain. The sojourn time in each pair of states of the bivariate Markov chain has geometric distribution like in any other Markov chain. Consecutive jumps of the primary process may occur while the secondary process has undergone several jumps along some path in the state space. Thus, the sojourn time of the primary process in each of its states is the sum of multiple independent geometric random variables. Considering different paths, we see that the sojourn time in each state of the primary process could have a mixture of convolutions of geometric distributions. A review of bivariate Markov chains may be found in [18].

Let $Z = \{Z_t = (X_t, S_t), t = 0, 1, \ldots\}$ denote a bivariate Markov chain where $X = \{X_t, t = 0, 1, \ldots\}$ represents the primary process and $S = \{S_t, t = 0, 1, \ldots\}$ represents the auxiliary process. We assume that X takes values in the state space $\mathbb{X} = \{0, 1, \ldots, d\}$, S takes values in the state space $\mathbb{S} = \{0, 1, \ldots, r\}$, and Z takes values in the state space $\mathbb{Z} = \mathbb{X} \times \mathbb{S}$. The state pairs $\{(a, i) \in \mathbb{Z}\}$ are assumed to be ordered lexicographically, and the transition probability of Z is given by

$$h_{ab}(ij) = P_\phi(Z_{t+1} = (b, j) \mid Z_t = (a, i)) \tag{4}$$

where ϕ is the parameter of the process, that is, ϕ comprises the set of independent entries of initial distribution and transition matrix of the bivariate Markov chain. The transition matrix $H = \{h_{ab}(ij)\}$ is written as a block matrix $H = \{H_{ab}; a, b \in \mathbb{X}\}$, where $H_{ab} = \{h_{ab}(ij); i, j \in \mathbb{S}\}$ is an $r \times r$ matrix. The underlying chain S is Markov with transition matrix Q if and only if the equation $\sum_{b \in \mathbb{X}} H_{ab} = Q$ holds independently of a. A similar condition can be given for the observable chain X to be Markov. When H is irreducible, it has a unique stationary distribution $\pi = \text{row}\{\pi_{ai}, a \in \mathbb{X}, i \in \mathbb{S}\}$ satisfying $\pi = \pi H$. The process $\{Z_t\}$ is stationary if and only if $P_\phi(Z_0 = (a, i)) = \pi_{a,i}$ for all $(a, i) \in \mathbb{Z}$.

The probability mass function of the sojourn time of the primary process in each state in $\mathbb{X}$ is given by [38]

$$p_\phi(l \mid a) = \bar{\nu}_a(\phi) H_{aa}^{l-1}(I - H_{aa})\mathbf{1} \tag{5}$$

for $l = 1, 2, \ldots$, where $\mathbf{1}$ is a column vector of all ones of suitable dimension, and $\bar{\nu}_a$ is defined as follows. Let $\nu_{a,i}(\phi) = P_\phi(Z_0 = (a, i))$ denote the initial probability of the bivariate Markov chain to be in state (a, i) at time $t = 0$, and let

$$\nu_a(\phi) = (\nu_{a,1}(\phi), \nu_{a,2}(\phi), \ldots, \nu_{a,r}(\phi)). \tag{6}$$

Then, $\bar{\nu}_a(\phi) = \nu_a(\phi)/(\nu_a(\phi)\mathbf{1})$ is a normalized version of $\nu_a(\phi)$. Equation (5) provides the *discrete-time phase-type* probability mass function with parameter $(\bar{\nu}_a(\phi), H_{aa})$ [37, p. 46]. This probability mass function is derived under the assumptions that the matrices H and $\{H_{aa}, a \in \mathbb{X}\}$ are irreducible and that the diagonal elements of H are positive.

Consider now substitution of the univariate Markov chain of an HMM with the bivariate Markov chain described above. The bivariate Markov chain is now observed through a memoryless channel with output denoted by $Y = \{Y_0, Y_1, \ldots, Y_t, \ldots\}$ as before. The resulting process (Y, X, S) is an HMM with a bivariate Markov chain $Z = (X, S)$ rather than the univariate Markov chain X. The sojourn time in each pair of states of the bivariate Markov chain is geometrically distributed. Assume now that the triplet process (Y, X, S) possesses a Markov property such that the processes Y and S are conditionally independent given the process X. With this assumption, the observable process Y inherits its sojourn time from the non-Markovian process X, rather than from the Markovian process Z, and hence, the sojourn time distribution of Y in each state of X is discrete phase-type rather than geometric [38], [18, Eq. 8.7]. The process (Y, X, S) with the Markovian property provides a realistic model for the received signal in a cognitive radio receiver [38]. We refer to the model that incorporates the above Markovian property as a *hidden bivariate Markov chain*.

Likelihood of Observable Process

Proceeding with the Markovian assumption, for each time instant t, Y_t is independent of S_t given X_t. That is, $b(y_t \mid z_t) = b(y_t \mid x_t)$. We next develop expressions for the likelihood function of the observable process and for the forward-backward formulas for both the HMM and the hidden bivariate Markov chain.

Define the conditional distribution

$$F_{ij}^{ab}(y) := P_\phi(Y_t \le y_t, Z_t = (b, j) \mid Z_{t-1} = (a, i)) \tag{7}$$

and the corresponding transition density

$$f_{ij}^{ab}(y_t) = \frac{\partial}{\partial y_t} F_{ij}^{ab}(y_t) = p_\phi(y_t, z_t = (b, j) \mid z_{t-1} = (a, i)). \tag{8}$$

Recall that the states (X, S) are ordered lexicographically. Define the $dr \times dr$ *transition density matrix* by

$$f(y_t) = \left\{ f_{ij}^{ab}(y_t); (a,i),(b,j) \in \mathbb{Z} \right\}. \tag{9}$$

The transition density matrix for the HMM where Y_t depends on Z_t is given in terms of the transition matrix H from (4) and the density which corresponds to $P_\phi(Y_t \leq y_t \mid Z_t = (b,j))$. We denote that density by $g_{\theta_{bj}}(y_t)$ where θ_{bj} is its parameter. This density is determined by the channel. For memoryless Gaussian channel as is assumed here, the parameter $\theta_{bj} = (\mu_{bj}, \sigma_{bj}^2)$, where μ_{bj} denotes the mean and σ_{bj}^2 denotes the variance. For the hidden bivariate Markov chain where Y_t given X_t is independent of S_t, the density $g_{\theta_{bj}}(y_t)$ is independent of j and is given by $g_{\theta_b}(y_t)$. In either case,

$$f(y_t) = HG(y_t) \tag{10}$$

where for the HMM,

$$G(y_t) = \mathrm{diag}(g_{\theta_{bj}}(y_t), (b,j) \in \mathbb{Z}), \tag{11}$$

and for the hidden bivariate Markov chain,

$$G(y_t) = \mathrm{diag}(g_{\theta_b}(y_t)I, b = 1, \ldots, d) \tag{12}$$

where I is an $r \times r$ identity matrix. Define the $1 \times dr$ row vector $\zeta_{y_0} = \{p_\phi(y_0, z_0); z_0 \in \mathbb{Z}\}$ representing the initial density of (Y_0, Z_0). Then, the likelihood function of the hidden bivariate Markov chain is given by

$$p_\phi(y_0^n) = \zeta_{y_0} \prod_{t=1}^{n} f(y_t)\mathbf{1}. \tag{13}$$

The likelihood function of the HMM is given by a similar expression. The parameter ϕ of the HMM comprises the independent components of the initial distribution π and of the transition matrix H and $\{\theta_{bj}, (b,j) \in \mathbb{Z}\}$. For the hidden bivariate Markov chain, the relevant entries of π and H are the same as for the HMM, but the parameter of the channel densities is given by $\{\theta_b, b \in \mathbb{X}\}$. The difference is due to the Markovian assumption making Y and S conditionally independent given X.

Forward-Backward Matrix Recursions

Evaluation of the likelihood function of the hidden bivariate Markov chain, as well as iterative estimation of its parameter using the EM algorithm, is facilitated by the use of the forward-backward recursions. We present here a slightly more general form of the standard recursions for HMMs due to Stiller and Radons [46]. This version is useful in recursive estimation of the parameter of the model [19]. Define

$$R(k,m) := \prod_{t=k}^{m} f(y_t) \tag{14}$$

where $1 \leq k \leq m \leq n$. For fixed k, we have the forward recursion (in m) on the backward density as follows:

$$\begin{aligned} R(m,m-1) &:= I \\ R(k,m) &= R(k,m-1) f(y_m) \end{aligned} \tag{15}$$

where I is an identity matrix. For fixed $m = n$, the backward recursion (in k) on the backward density is given as follows:

$$\begin{aligned} R(n+1,n) &:= I \\ R(k,n) &= R(k+1,n) f(y_k) \end{aligned} \tag{16}$$

for $k = n, n-1, \ldots, 0$. The forward recursion for the forward density is given by

$$L(m) = L(m-1) f(y_m) \tag{17}$$

where $m = 1, 2, \ldots, n$ and $L(m) = \nu_{y_0} R(1,m)$, and let $L(0) = \nu_{y_0}$. Note that for a given parameter ϕ and observation sequence $Y_0^n = y_0^n$, the $((a,i),(b,j))$ element of $R(k,m)$ is given by

$$R_{ai,bj}(k,m) = p_\phi(y_k^m, z_m = (b,j) \mid z_{k-1} = (a,i)), \tag{18}$$

and the (b,j) element of $L(m)$ is given by

$$L_{bj}(m) = p_\phi(y_0^m, z_m = (b,j)). \tag{19}$$

Numerical stability of the recursions in (15) and (17) is improved when scaling is introduced in each iteration. It is instructive to start with the description of the scaled version of $L(m)$, which we denote by $\tilde{L}(m)$. Let $c_0 = \nu_{y_0}\mathbf{1}$, and let $\tilde{L}(0) = \nu_{y_0}/c_0$. The scaled version of (17) is given by

$$\tilde{L}(m) = \frac{1}{c_m} \tilde{L}(m-1) f(y_m) \tag{20}$$

where

$$c_m = \tilde{L}(m-1) f(y_m)\mathbf{1}. \tag{21}$$

It follows that

$$\tilde{L}(m) = \frac{1}{\prod_{t=0}^{m} c_t} L(m), \tag{22}$$

$p_\phi(y_0^m) = \prod_{t=0}^{m} c_t$, $c_t = p_\phi(y_t \mid y_0^{t-1})$ for $t \geq 1$, and the (b, j) component of $\tilde{L}(m)$ is given by $\tilde{L}_{bj}(m) = P_\phi(Z_m = (b, j) \mid y_0^m)$. The scaled version of forward recursion on $R(k, m)$ is given by

$$\begin{aligned}\tilde{R}(k,m) &= \frac{1}{\prod_{t=k}^{m} c_t} R(k,m) \\ &= \tilde{R}(k, m-1)\frac{f(y_m)}{c_m}.\end{aligned} \tag{23}$$

where c_m is given in (21). The recursion $\tilde{R}(k,m)$ does not enjoy an appealing probabilistic interpretation as $\tilde{L}(m)$. A similar scaled backward recursion on $R(k,n)$ can be written.

The parameter of the hidden bivariate Markov chain may essentially be estimated as the parameter of an HMM using the Baum or the EM algorithm as is demonstrated in the next section "Estimation of the Bivariate Markov Chain Parameter." Batch estimation of the parameter from a given training sequence was detailed in [19, 38]. Sequential estimation of the parameter using an EM iterate was described in [19].

Estimation of the Bivariate Markov Chain Parameter

In this section we demonstrate how the parameter of the hidden bivariate Markov chain can be estimated from a sequence of observations y_0^n using the batch EM algorithm. Our presentation follows that in [19]. A sequential estimation approach which is based on the EM iteration may also be found in [19]. It turns out that the estimation procedure can be described simultaneously for an HMM as well as for a hidden bivariate Markov chain. We shall thus start with estimation of the HMM parameter and then infer about estimation of the hidden bivariate Markov chain.

Assume that ϕ_κ is the parameter estimate at the end of the κth iteration. At the conclusion of the $(\kappa + 1)$th iteration, the new estimate of the initial distribution π_{bj} is given by

$$\hat{\pi}_{bj}(n) = P_{\phi_\kappa}(Z_0 = (b, j) \mid y_0^n), \tag{24}$$

and the new estimate of $h_{ab}(ij)$ is given by

$$\hat{h}_{ab}(ij) = \frac{\hat{M}_{ij}^{ab}(n)}{\sum_{(\beta,l)} \hat{M}_{il}^{a\beta}(n)} \tag{25}$$

where $\hat{M}_{ij}^{ab}(n)$ denotes the conditional mean estimate given y_0^n of the number of transitions of Z from (a, i) to (b, j) in $[0, n]$. This number includes self-transitions.

Using the indicator function

$$\varphi_{bj}(t) = \begin{cases} 1, & Z_t = (b, j) \\ 0, & \text{otherwise,} \end{cases} \tag{26}$$

the number of transitions is given by

$$M_{ij}^{ab}(n) = \sum_{t=1}^{n} \varphi_{ai}(t-1)\varphi_{bj}(t), \tag{27}$$

and

$$\begin{aligned} \hat{M}_{ij}^{ab}(n) &= E_{\phi_\kappa}\left\{M_{ij}^{ab}(n) \mid y_0^n\right\} \\ &= \sum_{t=1}^{n} P_{\phi_\kappa}\left(Z_{t-1} = (a, i), Z_t = (b, j) \mid y_0^n\right). \end{aligned} \tag{28}$$

For an HMM with normal densities, $\{b_{\phi_\kappa}(y_t \mid z_t), z_t \in \mathbb{Z}\}$, with mean μ_{bj} and variance σ_{bj}^2 when $z_t = (b, j)$, define

$$N_j^b(n; \lambda) = \sum_{t=0}^{n} y_t^\lambda \varphi_{bj}(t) \tag{29}$$

for $\lambda \in \{0, 1, 2\}$. Note, for example, that if the observations $\{y_t\}$ are clustered into the various states of the HMM, then $N_j^b(n; 1)$ is the sum of the observations associated with state (b, j). Let

$$\begin{aligned} \hat{N}_j^b(n; \lambda) &= E_{\phi_\kappa}\left\{N_j^b(n; \lambda) \mid y_0^n\right\} \\ &= \sum_{t=0}^{n} y_t^\lambda P_{\phi_\kappa}(Z_t = (b, j) \mid y_0^n). \end{aligned} \tag{30}$$

The new estimates of the mean and variance at the conclusion of the $(\kappa + 1)$th iteration are, respectively, given by

$$\hat{\mu}_{bj}(n) = \frac{\hat{N}_j^b(n; 1)}{\hat{N}_j^b(n; 0)}, \tag{31}$$

$$\widehat{\sigma_{bj}^2}(n) = \frac{\hat{N}_j^b(n; 2)}{\hat{N}_j^b(n; 0)} - \hat{\mu}_{bj}^2(n). \tag{32}$$

For a hidden bivariate Markov chain, μ_{bj} and σ_{bj}^2 are reduced to μ_b and σ_b^2, respectively. Thus, $P_{\phi_\kappa}(Z_t = (b, j) \mid y_0^n)$ in (31) and (32) should be substituted by $P_{\phi_\kappa}(X_t = b \mid y_0^n)$. It is well known that the conditional probability in (28), and hence in (24) and (30), may be efficiently implemented using forward-backward recursions.

Spectrum Sensing Scenarios

In this section, we shall discuss the application of spectrum sensing techniques based on Markovian models to a variety of scenarios. Temporal spectrum sensing scenarios can be organized into three basic categories [47]:

1. *Narrowband:* The secondary user senses a single channel that is clearly defined in terms of center frequency and bandwidth.
2. *Multiband:* The secondary user senses multiple independent narrowband channels.
3. *Wideband:* The secondary user senses a spectrum band with no prior knowledge of channel boundaries or channel occupancy.

In the descriptions of the above categories, we "the secondary user" may refer to a group of several secondary users collaboratively sensing the given spectrum band. Spectrum sensing performance can be significantly enhanced by employing collaborative sensing among a group of several secondary users [30]. Multiband temporal sensing techniques are useful for applications such as TV whitespace where multiple independent primary users operate on clearly defined channels. To simplify our discussion, we shall assume that only a single primary user occupies a given narrowband channel. The wideband temporal sensing problem can be transformed into a multiband sensing problem by first identifying the channel boundaries [6].

Narrowband Sensing

Well-known signal detection algorithms for a narrowband channel include energy detection, cyclostationary feature detection, and matched filter detection [56]. The energy detector is the simplest of the narrowband detectors and requires no a priori knowledge of the channel, but performs poorly in low signal-to-noise ratio (SNR) conditions. The matched filter detector can detect primary user activity at very low SNR, but requires a priori knowledge of the primary user waveform. Cyclostationary feature detection lies between the matched filter and energy detector with respect to performance at low SNR, but requires significant computation times and long integration windows. The performance of all three signal detection methods can be degraded by low primary user duty cycle. By characterizing the primary user signal using a Markovian model, the temporal dynamics of primary user activity on the channel can be taken into account in detecting the signal. Spectrum sensing based on a Markovian model can lead to better detection performance in lower SNR scenarios, since the past history of primary user activity is incorporated into the detection process. Moreover, use of a Markovian model can provide predictive information that can be applied to achieve more effective dynamic spectrum access.

In [38], the hidden bivariate Markov chain model discussed in section "Bivariate Markov Chains" was proposed as a model for the received primary user signal in a narrowband channel. Consider a system consisting of one primary user transmitting on the narrowband channel. The primary user alternates between an active state, in which a signal of fixed power is transmitted over the narrowband channel, and an idle state, in which no signal is transmitted. We denote the idle state of the primary user at time $t\Delta$ by $X_t = 0$ and the active state by $X_t = 1$, where Δ is a sampling period. The process $X = \{X_t\}$, taking values in the set $\mathbb{X} \in \{0, 1\}$, models the directly observed state of the primary user.

The wireless propagation environment is assumed to be governed by a standard path loss with lognormal shadowing model [35, pp. 40–41]. We ignore fast fading since it can be reduced effectively by an averaging filter (cf. [34]). Let $u(t)$ denote the complex baseband demodulated primary user signal. For a particular secondary user, let $c(t)$ denote a random process representing the fading, and let $w(t)$ denote the additive thermal noise of the channel. The signal received by the secondary user is given by $y(t) = c(t)u(t) + w(t)$. The received baseband signal may be envisioned as a phasor perturbed by the additive noise. The received signal is sampled every Δ seconds, and each sample is represented by the logarithm of its power.

Let Y_t denote the logarithm of the power of the tth sample of the received signal of the secondary user. Given the state $X_t = a$ of the primary user, the samples $\{Y_t\}$ are assumed statistically independent, and each Y_t is assumed normally distributed with some mean μ_a and variance σ_a^2. The process $Y = \{Y_t\}$ represents the received primary user signal, which can be interpreted as the primary user state observed through the narrowband channel. Further motivation and discussion of this model for the cognitive radio channel is given in section "Hidden Markov Models."

If the primary user state process X is modeled as a Markov chain, the joint process (Y, X) is an HMM. In this case, the primary user sojourn times in the active and idle states are given by geometric distributions. We now introduce an underlying process S, taking values in $\mathbb{S} = \{0, 1, \ldots, r\}$, such that $Z = (X, S)$ is a bivariate Markov chain. In this case, the sojourn time of the process X in each state $a \in \{0, 1\}$ takes on a discrete-time phase-type distribution with r phases [38]. The trivariate process (Y, X, S) is then a hidden bivariate Markov chain. In the nomenclature of section "Bivariate Markov Chains," the conditional density of Y given $X = a$ is denoted by $g_{\theta_a}(y) = \mathcal{N}(\mu_a, \sigma_a^2)$, where $\mathcal{N}(\mu, \sigma^2)$ is the normal density with mean μ and variance σ^2. The parameter ϕ of the hidden bivariate Markov chain consists of the initial distribution of Z, denoted by $\boldsymbol{\pi}$, and the independent components of the generator of Z, denoted by H and $\{\theta_a; a \in \mathbb{X}\}$.

Assume that the parameter ϕ of the hidden bivariate Markov chain is given. The conditional probability of the bivariate state at time $t + \tau$ given the observations up to and including time t can then be computed as follows (cf. [38, Eq. (22)]):

$$p_\phi(z_{t+\tau} \mid y^t) = \sum_{z_t \in \mathbb{Z}} p_\phi(z_k \mid y^t)\; p_\phi(z_{t+\tau} \mid z_t)$$
$$= \sum_{z_t \in \mathbb{Z}} \tilde{L}_{z_t}(t)\, [H^\tau]_{z_t, z_{t+\tau}}\,, \tag{33}$$

where $[H^\tau]_{ai,bj}$ denotes the $((a,i);(b,j))$ entry of the generator matrix given by H^τ. A detection scheme for the state of the primary user at time $t+\tau$ given the received signal power y^t is specified by (cf. [38, Eq. (23)]):

$$\hat{X}_{t+\tau|t} = \begin{cases} 0, & \sum_s p_\phi(z_{t+\tau} = (0,s) \mid y^t) \geq \eta, \\ 1, & \text{otherwise}, \end{cases} \tag{34}$$

for $t = 0, 1, \ldots$, where η is a decision threshold, $0 < \eta < 1$. The detection scheme is a maximum a posteriori detector when $\eta = 0.5$. When $\tau = 0$, $\hat{X}_{t+\tau|t} = \hat{X}_{t|t}$ is an estimate of the current state X_t. When $\tau = 1, 2, \ldots$, $\hat{X}_{t+\tau|t}$ is the τ-step predicted estimate of the state $X_{t+\tau}$. The current and predicted state estimates $\hat{X}_{t+\tau|t}$ can be directly applied to make dynamic spectrum access decisions.

The parameter ϕ of the hidden bivariate Markov chain can be estimated offline from training data using the EM algorithm described in section "Estimation of the Bivariate Markov Chain Parameter"; see also [38]. A major advantage of online parameter estimation is that it can adapt to changes in the behavior of the primary user or the channel. An online approach to estimating the parameter, based on Rydén's recursive algorithm HMM parameter estimation, is developed in [48]. An alternative approach to online parameter estimation, based on the EM iteration discussed in section "Estimation of the Bivariate Markov Chain Parameter," is discussed in [19].

Collaborative Sensing

In radio environments with severe shadowing and fading effects, spectrum sensing by a single secondary user can lead to hidden terminal effects and other errors which can result in harmful interference to the primary users. Collaborative spectrum sensing techniques leverage multiuser diversity to improve sensing performance, particularly in severely shadowed environments with hidden terminals. Collaborative sensing involves multiple secondary users in a joint decision-making process to determine when a given channel is idle or active [30, 57]. Collaborative sensing schemes can be categorized into two main types: hard fusion and soft fusion. In this discussion, we review hard and soft fusion collaborative sensing and summarize the collaborative sensing schemes based on hidden bivariate Markov chain modeling developed in [49].

Hard Fusion

In a hard fusion scheme, at each time t, each secondary user q makes an independent decision, $X_{t|t}^{(q)}$, on the primary user state based on the observations $Y_1^{(q)}, \ldots, Y_t^{(q)}$. The 1-bit secondary user hard decisions are transmitted to the fusion center, which computes a final decision, denoted by $\hat{X}_{t|t}$, according to a hard fusion rule. For example, the "OR" rule decides that the primary user is active, i.e., state 1, if at least one of the secondary user hard decisions has the value 1. The "majority voting" rule decides that the primary user is active if more than half of the Q secondary user hard decisions have value 1. The OR rule and majority voting rule are special cases of the q-out-of-Q rule, where $1 \leq q \leq Q$ is an integer constant. Here, the primary user state is determined to be active if q or more of the individual hard decisions are "active"; otherwise, the primary user state is determined to be idle. The OR and majority voting rules are equivalent to the q-out-of-Q rule when $q = 1$ and $q = \lfloor Q/2 \rfloor$, respectively. The q-out-of-Q fusion rule is in turn a special case of linear hard fusion (cf. [41]). Under linear combining, the decision variable is computed as $V_t = \sum_{q=1}^{Q} w_q \hat{X}_{t|t}^{(q)}$, where the w_q are predetermined weights. The decision variable V_t is then compared to a threshold ψ to obtain the final decision $\hat{X}_{t|t}$. The q-out-of-Q fusion rule is a special case of linear hard fusion.

In conventional hard fusion schemes, each secondary user employs an energy detector to obtain a hard decision at each time t. Typically, a majority voting rule is applied at the fusion center. In the hard fusion scheme proposed in [49], each secondary user independently estimates the parameter of a hidden bivariate Markov chain to characterize the observed primary user signal on the channel. An estimator of the form (34) is employed to obtain a hard decision at the secondary user. The hard decisions are combined at the fusion center using a linear fusion rule proposed in [42] based on maximizing a so-called modified deflection coefficient.

Soft Fusion

In soft fusion, at each time t, the secondary users transmit quantized versions of their received signals $Y_t^{(1)}, \ldots, Y_t^{(q)}$, to the fusion center, where they are collectively used to predict the state of the primary user at time $t + \tau$ for some nonnegative integer τ. The state estimator is denoted by $\hat{X}_{t+\tau|t}$. For conventional fusion schemes that do not have predictive capability, $\tau = 0$. In a *linear* soft fusion scheme, a weighted sum, $V_t = \sum_{q=1}^{Q} w_q Y_t^{(q)}$, of the observations at time k is computed and compared to a threshold ψ as follows [33, 41, 42]:

$$\hat{X}_{t|t} = \begin{cases} 0, & V_t < \psi, \\ 1, & V_t \geq \psi, \end{cases} \tag{35}$$

where $w_1, \ldots, w_Q$ are the weights. Typically, the threshold and weights for soft fusion are computed offline [33, 42].

In [48], at each time t, the observation sample $Y_t^{(q)}$ from each secondary user q is transmitted directly to the fusion center, which forms the vector observation sample $\boldsymbol{Y}_t = (Y_t^{(1)}, \ldots, Y_t^{(Q)})$. The soft fusion scheme is based on hidden bivariate Markov chain modeling of the *vector* observation sequence $\boldsymbol{Y} = \{\boldsymbol{Y}_t; t = 0, 1, \ldots\}$ generated by the Q secondary users. Here, the conditional output density parameter is given by $\boldsymbol{\theta} = (\boldsymbol{\theta}_a : a \in \mathbb{X})$, where $\boldsymbol{\theta}_a = (\theta_a^{(1)}, \ldots, \theta_a^{(Q)})$, and $\theta_a^{(q)}$ is the conditional output density parameter of each secondary user q when the primary user is in state a. With this definition of $\boldsymbol{\theta}$, the parameter ϕ of the hidden bivariate Markov chain is given by the independent elements of $(\pi, \boldsymbol{\theta}, H)$, where π is the initial state probability distribution and H is the generator of the underlying bivariate Markov chain.

Parameter estimation of the hidden bivariate Markov chain with vector observation input $\boldsymbol{Y}$ can be carried out at the fusion center using the EM approach discussed in section "Estimation of the Bivariate Markov Chain Parameter" by replacing the scalar sequence y^t with the vector sequence $\boldsymbol{y}^t$ and interpreting the parameter ϕ as discussed above. Similarly, state estimation can be performed via (34). The online parameter estimation approach in [48] can be extended to handle vector observation input, as described in [49].

Performance Comparison

Receiver operating characteristic (ROC) curves presented in [49] show a significant improvement in detection performance of collaborative sensing schemes based on the hidden bivariate Markov chain compared to conventional hard and soft fusion schemes based on energy detectors. In particular, the Markovian model-based soft fusion scheme performs markedly better than the linear soft fusion-based scheme proposed in [42]. Linear soft fusion performs better than the hard fusion scheme based on hidden bivariate Markov chain modeling, which in turn performs substantially better than conventional hard fusion based on energy detection. A disadvantage of soft fusion schemes is that they incur substantially higher communication overhead than the hard fusion schemes. This overhead can be reduced by quantizing the received signal strength values using a smaller number of bits at the expense of poorer detection accuracy, as proposed in [49]. Using a simple uniform quantization scheme, soft fusion based on the hidden bivariate Markov chain with 4-bit observation samples was shown to outperform linear soft fusion with 8-bit samples.

Multiband Sensing

In multiband spectrum sensing, the secondary user tracks the states of primary users operating on a given set of channels to determine spectrum access opportunities. The center frequency and bandwidth of each channel are assumed known. In a given sensing interval, the secondary user must allocate time for sensing each of the channels. A Markovian model for multiband sensing was proposed by [50], in which each primary user on a given channel is modeled as a two-state homogeneous continuous-time Markov chain. Hence, the state of the primary user on each channel is assumed to be observable directly, or at least the signal-to-noise ratio is sufficiently high that sensing errors are negligible. The Markov chains corresponding to different primary users are assumed statistically independent. One of the basic issues in multiband sensing involves how much time should be allocated to sensing each channel.

To discuss this model further, let us assume there are M independent channels, each having the same bandwidth, but the primary user model parameters for the channels may be different. The parameter of each Markov chain is not known in advance and hence is estimated from observations of the state processes. In a given sensing interval of length T seconds, the secondary user senses each channel i for T_i seconds, where $\sum_{i=1}^{M} T_i = T$. In [50], the sensing times $\{T_i\}$ are determined by minimizing the Cramer-Rao lower bound on the minimum mean squared error in estimating the parameters of *all* M channels. An approximation for the inverse Fisher information matrix, asymptotic in the sensing interval length T, is used to obtain closed-form formulas for the MMSE sensing time allocations.

Since not all M channels may provide equally *good* secondary user spectrum access opportunities, allocating channel sensing times with the objective of minimizing the overall estimation error may not be optimal for dynamic spectrum access. In [8], a preprocessing step to determine the *best* $N < M$ channels, with respect to a criterion related to the spectrum opportunity on the channel, is performed first. The criterion used in [8] was based on the mean idle time on the channel. Then the approach of [50] is applied to the remaining N channels from the first step. The preprocessing step in [8] is based on the optimal computing budget allocation (OCBA) methodology [12] from the field of simulation optimization. The OCBA approach was originally developed to test multiple designs through simulation by allocating simulation time to the designs with the objective of maximizing the probability that the best design is selected according to a given cost function under a Gaussian model [13]. The technique was subsequently extended to determine the best $N > 1$ designs among a given set of M designs [14]. In the context of multichannel parameter estimation, sensing times are allocated rather than simulation times, and the multiple designs correspond to the multiple channels in multiband sensing.

A number of articles on multiband spectrum sensing have approached the problem as a type of multiarmed bandit problem [3,39,53,59] or the related partially observable Markov decision process (POMDP) [58]. Several assume knowledge of the parameters of the underlying Markov chains, but do not address the important issue of parameter estimation [3, 53, 58]. The multichannel parameter estimation algorithm proposed in [8] obtains estimates of this parameter and thus could, in principle, be used in conjunction with these approaches. Knowledge of the model parameter can be used to improve spectrum detection performance and allows the prediction of future primary user state, which provides clear advantages for spectrum sensing [38, 53].

Wideband Sensing

In the wideband spectrum sensing scenario, a secondary user must sense an entire band and determine channel boundaries. The bandwidth that must be sensed can vary from the order of 1 MHz to 1 GHz. This is required if the secondary user cannot leverage any external information about channel allocation. A secondary user need only perform wideband sensing during initialization and may then revert to multiband or narrowband sensing during normal operation. In general, primary user signals may be heterogeneous in frequency, bandwidth, and power, so robust wideband sensing algorithms must be developed to detect all primary user activity within the spectrum band.

State-of-the-art techniques for wideband sensing include wideband energy detection [9] and frequency-domain edge detection [51]. The wideband energy detector is a very simple wideband sensing technique in which the secondary user estimates the power spectral density over the entire band and applies an energy threshold to determine primary user activity [9,25]. Many power spectral density frames may be averaged to increase reliability. This simple algorithm has several limitations. Like all energy detectors in additive white Gaussian noise (AWGN), this technique has limited sensitivity, and performance is severely degraded at low SNR. Furthermore, this technique operates on a snapshot in time, and dynamic behavior of the primary user will degrade performance, since both the on and off cycles will be averaged into the power spectral density estimate.

Edge detectors can offer an improvement over energy detection in terms of SNR threshold, but they can also perform relatively poorly on signals with gradual roll-offs in their band edges. An alternative wideband spectrum sensing technique that has been studied in the literature employs frequency-domain edge detection to determine channel boundaries. A popular edge detection technique uses the continuous wavelet transform to decompose the edge detector into multiple resolutions and multiplies the resolutions together, which has a beneficial effect of reducing the noise [51]. While the edge detectors do offer an improvement over energy detectors in terms of SNR threshold, they come with several limitations. Most importantly, the edge detectors require that primary user signals have sharp transitions in the frequency domain. This allows them to work well with the rectangular spectra of OFDM and quadrature amplitude modulation (QAM) with low excess bandwidth, but edge detectors tend to fail on signals with gradual roll-offs on their band edges, such as QAM with large excess bandwidth and GMSK.

Neither energy detection nor edge detection alone takes into account the temporal dynamics of primary user signals and consequently can perform rather poorly when primary user signals have low duty cycles. In [6], a framework for wideband temporal spectrum sensing is proposed in which a given spectrum band is divided into smaller channels and modeled as a balanced binary tree. In [6], a sensing framework for reliable wideband detection of primary users with low duty cycle was developed. The approach, referred to as wideband temporal sensing, involves partitioning the given spectrum band into smaller subchannels. The energy in each subchannel is measured and an HMM-based spectrum sensing approach is applied to each subchannel. A recursive tree search is performed to aggregate correlated subchannels into a set of independent narrowband channels, which effectively reduces the sensing task to the multiband case. The wideband temporal sensing approach developed in [6] allows primary user signals with low duty cycle to be detected accurately at high to moderate SNR. This approach was demonstrated to outperform both wideband energy and edge detection techniques particularly in the presence of dynamic primary user signals. In principle, the recursive tree search could be based on a hidden bivariate Markov chain in lieu of the HMM to provide more accurate modeling of the state sojourn times.

In [7], the wideband temporal sensing approach of [6] is extended to incorporate edge detection. In particular, the wavelet-based edge detection algorithm in [51] is incorporated into the wideband temporal sensing framework of [6]. The use of edge detection avoids the need for the recursive tree search used in the wideband temporal energy detector, resulting in a computationally more efficient spectrum sensing scheme. Moreover, the wideband temporal edge detector was shown to perform better in low SNR scenarios in the presence of primary user signals with sharp band edges, i.e., OFDM signals.

Research Challenges and Open Problems

In this section, we discuss future challenges and open research problems related to spectrum sensing using Markovian models.

Signal Feature vs. Energy Detection

In the Markovian model-based spectrum sensing techniques discussed in section "Spectrum Sensing Scenarios," the front end for spectrum sensing was assumed to be similar to that of an energy detector. The energy of the received signal samples was measured and used to perform parameter estimation of a hidden bivariate Markov chain, as well as detection and prediction of primary user spectrum usage. The energy-based front end could, in principle, be replaced by a front end that extracts certain features of the received signal. This could potentially improve the performance of Markovian-based spectrum sensing in low SNR scenarios.

Cyclic feature detection, for example, is based on computing the cyclic or cyclostationary spectrum of the received signal. Modulated signals are often readily identifiable based on their cyclic spectra. Therefore, cyclic feature detection has been proposed as an approach to spectrum sensing that has the potential to perform much better than energy detection or edge detection in low SNR scenarios [16, 31, 32]. A drawback of cyclic feature detection is the usual requirement to sample above the Nyquist rate to recover the cyclic spectrum. Furthermore, long sensing periods are typically required to obtain a statistically reliable estimate of the cyclic spectrum [24]. A cyclic feature detector also has much higher computational complexity compared to an energy detector. Nevertheless, practical implementation of cyclic feature detection for spectrum sensing is the subject of ongoing research. Aside from the cyclic spectrum, other features of the received signal could be used to perform spectrum sensing based on Markovian models. For example, the cepstrum has been used successfully in speech recognition in conjunction with HMMs [21, 43].

Multiband Sensing with Channel Impairments

Multiband sensing was discussed in section "Multiband Sensing" mainly in the context of determining appropriate sensing intervals for each channel, based on the framework of [50]. In [8, 50], the primary user activity on each channel is modeled by a two-state continuous-time Markov chain in which the primary user state was assumed to be observed directly. In practice, the primary user state is observed only through the received signal, which is subject to channel impairments such as fading, shadowing, and additive white Gaussian noise. Therefore, it would be of interest to extend the approach to incorporate channel impairments. Some form of noise could be incorporated into the continuous-time Markov primary user state model. Alternatively, a multiband sensing approach based on the HMM or hidden bivariate Markov model in discrete time could be developed.

Multiuser Channels

In the temporal spectrum sensing scenarios discussed in section "Spectrum Sensing Scenarios," a single primary user was assumed to occupy a given narrowband channel. More generally, multiple primary users could share a given channel in a dynamic spectrum sensing scenario. In this case, the two-state Markov model discussed previously would not be sufficient to characterize the channel. One approach is to increase the number of states in the model. For example, if there were N distinct primary users sharing a channel, with different received signal characteristics, an $(N+1)$-state Markov channel could be used to model the channel. An alternative approach is to introduce a Gaussian mixture model for the primary user state. In this case, the Markov model would still consist of two states, i.e., an active and an idle state, but the conditional distribution in the active state would be governed by a mixture of N Gaussian random variables.

Wideband Sensing

The wideband temporal spectrum sensing approach discussed in section "Wideband Sensing" has limitations with respect to the spectrum bandwidth that can be sensed in a practical implementation. This approach could be extended to cover larger spectrum bands by partitioning the spectrum into smaller chunks and then employing high-performance hardware to implement wideband temporal sensing in parallel over the spectrum chunks. However, for very wide spectrum bands, e.g., on the order of tens or hundreds of GHz, the required hardware may be too expensive in terms of actual cost and/or power consumption.

Detection of temporal spectrum hole opportunities over a wide spectrum band remains an open problem. In general, spectrum sensing over very wide spectrum bands requires prohibitively high sampling rates for current analog to digital converters. A drawback of cyclic feature detection is the usual requirement to sample above the Nyquist rate to recover the cyclic spectrum. Compressive sensing has been proposed as a technique to exploit the sparsity of primary user signal occupancy within a wide spectrum band. In [52], the inherent sparsity in the two-dimensional cyclic spectrum of communication signals is leveraged, and compressive sensing and sparse signal techniques are developed to recover the desired cyclic statistics with sub-Nyquist samples in a robust manner. Related work along these lines is reported in [36].

Wideband spectrum sensing techniques based on compressive sensing do not take into account the bursting nature of primary user signals, which may result in false detection of spectrum holes in the spectrum band. Moreover, compressive sensing techniques provide only a snapshot of primary user activity over the given spectrum and do not provide a mean of exploiting temporal spectrum hole opportunities. In principle, the primary user activity over the spectrum band during a given time slot could be characterized by a state, and the temporal dynamics could then be modeled using a Markov chain. Unfortunately, the number of states required for such a Markovian model would be infeasible for practical implementation.

A possible solution may involve a two-stage approach in which wideband sensing techniques such as those based on compressive sensing may be applied in the first stage to provide a rough picture of the spectrum occupancy at a given epoch. Portions of the spectrum may then be characterized as being idle, fully occupied, or partially occupied. In the second stage, the Markovian-based wideband temporal sensing approach may be applied to the portions of the spectrum deemed partially occupied based on the results of the first stage. A partially occupied spectrum band may be classified as such because of bursting primary user signal activity, or simply because of a weak primary user signal. In the former case, temporal spectrum hole opportunities may exist, whereas in the latter case, a spatial spectrum hole opportunity may exist. Clearly, these cases must be treated differently in a dynamic spectrum access or dynamic spectrum sharing systems.

Resource Allocation

In this chapter, we have focused on Markovian-based spectrum sensing methods to detect and predict spectrum hole opportunities in the settings of a narrowband channel, a multiband scenario, and a wideband scenario. Emphasis has been placed on estimating the parameter of a Markovian model to characterize primary user occupancy in a given spectrum band. Given an estimate of the model parameter, detection and prediction of the primary user state for spectrum sensing follows from the Markovian model. On the other hand, we have not discussed the important issue of how to allocate harvested spectrum resources to the secondary users. In the language of multiarmed bandit problems, our focus has been on *exploration* rather than *exploitation* of the spectrum.

The spectrum sensing techniques we have discussed may be employed by a spectrum monitoring service. In this scenario, the secondary users may contribute observation data to the spectrum monitoring service, but they do not make the spectrum sensing decisions per se. The secondary users access the spectrum holes by making requests to the spectrum monitoring service. In this way, exploration of the spectrum is decoupled from its exploitation by secondary users. In this setting, the spectrum monitoring service is responsible both for making spectrum sensing decisions and spectrum allocation decisions to the secondary users. An interesting problem requiring further research is how the Markovian characterization of primary user spectrum occupancy can be leveraged to perform spectrum allocation to secondary users in an efficient manner. In the multiband scenario, for example, when multiple channels are detected to be idle, a secondary user requesting a channel for dynamic spectrum access may be assigned the best channel with respect to some criterion such as the expected sojourn time in the idle state. Such resource allocation could also depend on further information provided by the secondary user, for example, the expected length of time that the channel would be needed. Furthermore, the transmission requirements of secondary users may play a role in which portions of the spectrum are sensed by the spectrum monitoring service.

The presence of a spectrum monitoring service effectively decouples the exploration task from the exploitation task in dynamic spectrum access or sharing. When the secondary users are responsible for both tasks, an important trade-off between exploration and exploitation arises. As discussed in section "Background and Overview of Related Work," several works in the literature have addressed this issue by modeling the dynamic spectrum access problem as POMDP or multiarmed bandit problem. These problems are formulated in a multiband setting in which the the underlying model of primary user activity for a given channel is assumed to be Markovian. A given secondary user must decide, in a given time slot, whether to perform sensing, i.e., explore, or to access a channel, i.e., exploit. If it decides to explore, the secondary user must further decide which channel or channels to sense. Similarly, if it decides to exploit, the secondary user must decide which channel or channels to exploit. In the POMDP formulations, the parameter of the underlying Markov process is assumed to be known either a prior or via parameter estimation. In the multiarmed bandit formulations, usually full knowledge of the parameter is not needed.

In the OCBA-based approach to multiband sensing proposed in [8], the N *best* channels out of a total M channels is selected, based on a parameter estimation process. In the context of a secondary user implementing a POMDP or multiarmed bandit approach to joint sensing and resource allocation, this approach is particularly beneficial when $N \ll M$. Reducing the number of channels under consideration from N to M can have a significant impact on the computational burden. Moreover, the parameter estimates for the M channels can be used in both the POMDP and multiarmed bandit approaches. It would be of interest to investigate further the role that parameter estimation could play with respect to the exploration/exploitation trade-off in the multiband setting.

Conclusion

In this chapter, we have discussed the application of Markovian models to spectrum sensing in cognitive radio networks. The focus of the chapter has been on the formulation of the Markovian models, parameter estimation, and detection/prediction of primary user activity in a given spectrum band based on knowledge of the model parameter. We provided an overview of background material and related work on spectrum sensing using Markovian models. We reviewed relevant Markovian models for spectrum sensing, primarily in discrete time, including Markov chains, hidden Markov models, and multivariate Markov chains. We also reviewed parameter estimation techniques for these models, particularly the Baum algorithm. We then discussed the application of Markovian-based spectrum sensing in the three progressively more difficult settings of narrowband, multiband, and wideband sensing. Finally, we discussed some interesting issues and open problems for further research on spectrum sensing using Markovian models.

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