

Incorporating Multi-Source Transfer Learning into an Unscented Kalman Filter for Object Tracking

Omar Alotaibi and Brian L. Mark
Dept. of Electrical and Computer Engineering
George Mason University
Fairfax, Virginia, U.S.A.
oalotaib@gmu.edu, bmark@gmu.edu

Abstract—We develop a framework for incorporating Bayesian transfer learning into an unscented Kalman filter (UKF) to track a nonlinear dynamic motion model in a multi-source system consisting of two or more sensors. This framework extends a previous UKF-based transfer learning scheme for object tracking in a two-sensor system. Our objective is to extend the application of transfer learning to leverage knowledge gained from several source sensors. To achieve this aim, we generalize the transfer learning approach to online multi-source Bayesian transfer learning. The extended framework is approximated via a UKF, where the predicted observation densities of one or more sources are transferred to a primary sensor experiencing higher measurement noise intensity. By leveraging knowledge from multiple sources instead of a single source, the tracking performance of the primary sensor is able to achieve a higher level of estimation accuracy. The performance gain increases proportionally with the number of source sensors. Our numerical results demonstrate the effectiveness of multi-source Bayesian transfer learning in improving tracking accuracy.

Index Terms—Multi-source transfer learning, object tracking, Bayesian transfer learning, unscented Kalman filter.

I. INTRODUCTION

Object tracking using a single sensor or multiple sensors has been widely used in various applications. In a practical multiple sensor tracking system, sensors with overlapping field-of-views experience diverse environmental conditions that can negatively impact their observed measurements. A given sensor within the multiple sensor system may encounter rougher conditions, with higher noise intensity compared to the other sensors, resulting in less accurate object tracking estimates. In a real-world scenario, multiple operators might manage classified tracking systems, each configured differently from others in the same coverage area. Developing a collaborative technique among these operators to enhance a tracking system negatively impacted by higher noise intensity poses security and privacy challenges. These challenges can be addressed by limiting the information shared among the systems in order to avoid the risks of reverse engineering, jamming, or blind spots identification. In this paper, we investigate transfer learning as an approach to limited information sharing in a multi-sensor system that can enhance tracking performance.

Bayesian transfer learning (BTL) [1], [2], has recently been introduced into object tracking. This technique leverages previously gained knowledge between sensors with the aim of achieving lower loss of accuracy in the estimation process.

In [3], BTL was incorporated with Kalman filter (KF) [4] for tracking an object following a linear model via a *dual-sensor* tracking system. In the context of nonlinear models, local approximation approaches, particularly UKF [5] and cubature Kalman filter (CKF) [6], were successfully adopted BTL in [7], [8] for enhancing the tracking performance of the primary sensor in a two-sensor system. The transfer learning approach was extended to learn and extract knowledge from multiple sources in [9]. In the field of deep learning, the use of multi-source knowledge for a classifier with domain adaptation has been shown to achieve superior accuracy performance compared to the adoption of a single source domain [10]. Moreover, online multi-source transfer learning (OMTL) has been applied to enhance classification performance in the context of homogeneous and heterogeneous domains in [11]. Recently, a model-based method was proposed to predict the trajectory of pedestrians with OMTL allowing the model to learn quickly from the new scene [12]. It is important to emphasize that the BTL approach differs from the distributed Kalman filter (DKF) [13], [14], which has been employed in sensor networks. The DKF has higher communication overhead due to the sharing of estimated states, higher-dimensional data, and observed measurements, including clutter, among all sensors. Furthermore, from a communications perspective, the DKF incurs a one time-step delay in the filtering process.

The present paper proposes a generalization of Bayesian transfer learning filter (BTLF) for the dual-sensor tracking system, to formulate a *multi-sensor* tracking system with potentially more than two sensors, referred to as an online multi-source Bayesian transfer learning filter (OMBTLF), in which knowledge from multiple source sensors is simultaneously transferred and leveraged into the primary sensor tracking filter. The UKF is integrated into the OMBTLF framework to track an object following a nonlinear trajectory. The generalized framework incorporates knowledge from multiple sources and integrates predicted observation densities parameters, means and covariances from these sources within the primary sensor filtering process. Our simulation results demonstrate that the accuracy performance of the primary tracking filter increases proportionally as the number of sources increases. As expected, a larger number of sources provides the primary sensor leveraging with more gained knowledge into the filtering process compared to incorporating

knowledge from a single source.

The remainder of this paper is structured as follows. Section II introduces notation and describes the representation of the discrete nonlinear dynamic system. Section III gives an overview of the OMBTLF framework and formulates the problem of transfer learning in a multi-sensor system. In Section IV, we develop our proposed UKF-based multi-sensor tracking scheme with transfer learning. Our numerical results and findings are detailed in Section V. The paper is concluded in Section VI.

II. PRELIMINARIES

A. Notation

Throughout this paper, we use uppercase and lowercase letters for scalars, uppercase calligraphic and bold lowercase letters for vectors, bold uppercase letters for matrices, and bold uppercase calligraphic letters for sets. The transpose, inverse, and diagonal of a matrix are denoted as $(\cdot)^T$, $(\cdot)^{-1}$, and $\text{diag}[\cdot]$, respectively. The sets of natural, real, and positive real numbers are denoted by \mathbb{N} , \mathbb{R} , and \mathbb{R}^+ , respectively.

B. Nonlinear Dynamic System

In this paper, we consider a discrete time dynamic system for a single moving object following a nonlinear model represented via a state space as

$$\mathbf{x}_k = f_k(\mathbf{x}_{k-1}) + \mathbf{v}_{k-1}, \quad (1)$$

$$\mathbf{z}_k = h_k(\mathbf{x}_k) + \mathbf{w}_k, \quad (2)$$

where $k \in \mathbb{N}$ represents the discrete time step of the dynamical motion. The n_x -dimensional state vector of the moving object is denoted by $\mathbf{x}_k \in \mathbb{R}^{n_x}$ and its motion is governed by a nonlinear transition function $f: \mathbb{R}^{n_x} \rightarrow \mathbb{R}^{n_x}$. The observed measurement is represented by an n_z -dimensional vector $\mathbf{z}_k \in \mathbb{R}^{n_z}$, which is determined by a nonlinear measurement function $h: \mathbb{R}^{n_x} \rightarrow \mathbb{R}^{n_z}$, which maps the state vector into the measurement space [15]. The noise vectors of the process and measurement that identify uncertainty and disturbance in the state and measurement models are represented as $\mathbf{v}_{k-1} \in \mathbb{R}^{n_x}$ and $\mathbf{w}_k \in \mathbb{R}^{n_z}$, respectively.

III. ONLINE MULTI-SOURCE BAYESIAN TRANSFER LEARNING FILTER APPROACH

A. Problem Statement

Consider a multi-sensor system with $L + 1$ sensors tracking a moving object represented by the dynamical motion model in (1). The process noise is assumed to be an independent, identically distributed (iid), zero-mean Gaussian noise process $\{\mathbf{v}_k : k \in \mathbb{N}\}$, where $\mathbf{v}_k \stackrel{\text{iid}}{\sim} \mathcal{N}(\mathbf{0}, \mathbf{Q}_v)$ with identical associated covariance $\mathbf{Q}_v \in \mathbb{R}^{n_x \times n_x}$ for all sensors. The $L + 1$ sensors in the multi-sensor system observe measurements with various measurement noise intensities due to the surrounding environmental conditions that each sensor in the multi-sensor system imposes. As shown in Fig. 1, we assume that a single sensor in the system, referred to as the primary sensor, experiences higher noise intensity and observes measurements

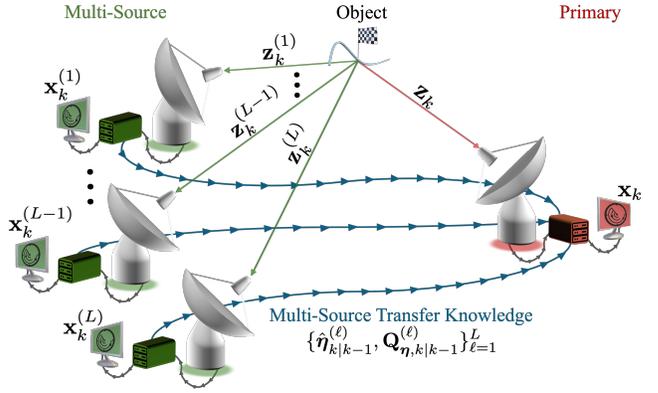


Fig. 1. Illustration of a tracking system involving L multi-source sensors engaged in online multi-source transfer learning.

represented by $\mathbf{z}_k = h(\mathbf{x}_k) + \mathbf{w}_k$. The remaining L sensors are referred to as multi-source sensors. The measurement of the ℓ th multi-source sensor is given by $\mathbf{z}_k^{(\ell)} = h(\mathbf{x}_k) + \mathbf{w}_k^{(\ell)}$, where $\ell = 1, \dots, L$. The measurement noises of the primary and the ℓ th source sensor is assumed to be zero-mean Gaussian noise given, respectively, by $\mathbf{w}_k \stackrel{\text{iid}}{\sim} \mathcal{N}(\mathbf{0}, \mathbf{Q}_w)$ and $\mathbf{w}_k^{(\ell)} \stackrel{\text{iid}}{\sim} \mathcal{N}(\mathbf{0}, \mathbf{Q}_w^{(\ell)})$, where the corresponding covariances $\mathbf{Q}_w \in \mathbb{R}^{n_z \times n_z}$ and $\mathbf{Q}_w^{(\ell)} \in \mathbb{R}^{n_z \times n_z}$ are characterized by

$$\mathbf{Q}_w = I_w \mathbf{B}_w \text{ and } \mathbf{Q}_w^{(\ell)} = I_w^{(\ell)} \mathbf{B}_w \quad (3)$$

with noise intensities $I_w \in \mathbb{R}^+$ and $I_w^{(\ell)} \in \mathbb{R}^+$ and common matrix $\mathbf{B}_w \in \mathbb{R}^{n_z \times n_z}$. The primary sensor has a higher noise intensity than of the L multi-source sensors, i.e., $I_w > I_w^{(\ell)}$, due to the more challenging environment that it experiences. To resolve this issue, we apply the OMBTLF approach to extract predicted observation knowledge from L multi-source sensors and then transfer this knowledge to the primary sensor, as we detail in the following sections.

B. Multi-Source Tracking Filter

We extend the BTLF framework introduced in [7], [8] to accommodate two or more multi-source sensors instead of a single source. The multi-source tracking filter is assumed to track a single object, although each of the L multi-sources observes its own set of measurements. For the ℓ th source tracking filter, given a set of measurements $\mathbf{z}_{1:k}^{(\ell)}$, an unknown object state $\mathbf{x}_k^{(\ell)}$ and predicted observation $\eta_{k+1}^{(\ell)}$ are estimated according to the overall posterior density obtained as

$$p(\mathbf{x}_k^{(\ell)}, \eta_{k+1}^{(\ell)} | \mathbf{z}_{1:k}^{(\ell)}) \propto p(\mathbf{x}_k^{(\ell)} | \mathbf{z}_{1:k}^{(\ell)}) p(\eta_{k+1}^{(\ell)} | \mathbf{z}_{1:k}^{(\ell)}), \quad (4)$$

where the first term denotes the posterior density of the object state, which is estimated through the ℓ th source tracking filter as follows:

$$p(\mathbf{x}_k^{(\ell)} | \mathbf{z}_{1:k}^{(\ell)}) \propto p(\mathbf{z}_k^{(\ell)} | \mathbf{x}_k^{(\ell)}) p(\mathbf{x}_k^{(\ell)} | \mathbf{x}_{k-1}^{(\ell)}) p(\mathbf{x}_{k-1}^{(\ell)} | \mathbf{z}_{1:k-1}^{(\ell)}). \quad (5)$$

The posterior density of the predicted observation to extract the knowledge, as the predicted observation of the next time step, of the ℓ th source is expressed as

$$p(\boldsymbol{\eta}_{k+1}^{(\ell)} | \mathbf{z}_{1:k}^{(\ell)}) \propto p(\boldsymbol{\eta}_{k+1}^{(\ell)} | \mathbf{x}_{k+1}^{(\ell)}) p(\mathbf{x}_{k+1}^{(\ell)} | \mathbf{x}_k^{(\ell)}) p(\mathbf{x}_k^{(\ell)} | \mathbf{z}_{1:k}^{(\ell)}), \quad (6)$$

where $p(\mathbf{x}_{k+1}^{(\ell)} | \mathbf{x}_k^{(\ell)})$ and $p(\boldsymbol{\eta}_{k+1}^{(\ell)} | \mathbf{x}_{k+1}^{(\ell)})$ follow the state and measurement models of the nonlinear dynamic system represented in (1) and (2), respectively. The predicted observation of each multi-source is obtained and transferred simultaneously to the primary tracking filter.

C. Primary Tracking Filter

The primary tracking filter collects predicted observations transferred from multiple sources, resulting in a set of collected predicted observations, denoted as $\mathcal{H}_{2:k} = \{\boldsymbol{\eta}_{2:k}^{(1)}, \dots, \boldsymbol{\eta}_{2:k}^{(L)}\}$. Given a set of observed measurements $\mathbf{z}_{1:k}$ of the primary sensor and leveraging the set of transferred predicted observations $\mathcal{H}_{2:k}$ from multiple sources, the posterior density of the unknown object state satisfies

$$p(\mathbf{x}_k | \mathbf{z}_{1:k}, \mathcal{H}_{2:k}) \propto p(\mathbf{z}_k | \mathbf{x}_k) p(\mathbf{x}_k | \mathbf{z}_{1:k-1}, \mathcal{H}_{2:k}), \quad (7)$$

where the second term in (7) defines the density of the object state that is updated via leveraging transferred knowledge from L sources as an intermediate step, prior to updating step through the measurement observed by the primary sensor. Conditioning on the same object state \mathbf{x}_k , the predicted observations of each source are assumed to be conditionally independent, the density $p(\mathbf{x}_k | \mathbf{z}_{1:k-1}, \mathcal{H}_{2:k})$ is given by

$$p(\mathbf{x}_k | \mathbf{z}_{1:k-1}, \mathcal{H}_{2:k}) \propto \prod_{\ell=1}^L p(\boldsymbol{\eta}_k^{(\ell)} | \mathbf{x}_k) p(\mathbf{x}_k | \mathbf{z}_{1:k-1}, \mathcal{H}_{2:k-1}). \quad (8)$$

As described in (7) and (8), the overall posterior density of the object state in the primary tracking filter is estimated using the observed measurement and transferred knowledge leveraged from multiple sources.

IV. OMBTL FOR UNSCENTED KALMAN FILTER

The UKF [5] is approximated through a set of N_p weighted sigma points, $\{\mathcal{X}_j, W_j\}_{j=1}^{N_p}$. Consider an n_x -dimensional state density at time step k following a Gaussian distribution defined as $\mathbf{x}_k \sim \mathcal{N}(\hat{\mathbf{x}}_{k|k}, \mathbf{P}_{k|k})$. The sigma points of the state density are obtained using the following rule.

- *Unscented Transformation*: The sigma points computed via the unscented transformation (UT) in [16] comprise $N_p = 2n_x + 1$ sigma points defined as

$$\mathcal{X}_{j,k|k} = \begin{cases} \hat{\mathbf{x}}_{k|k} + \sqrt{(n_x + \lambda) \mathbf{P}_{j,k|k}}, & j = 1, 2, \dots, n_x, \\ \hat{\mathbf{x}}_{k|k} - \sqrt{(n_x + \lambda) \mathbf{P}_{j-n_x,k|k}}, & j = n_x + 1, \dots, 2n_x, \\ \hat{\mathbf{x}}_{k|k}, & j = 2n_x + 1, \end{cases} \quad (9)$$

where the scaling parameter is $\lambda = \alpha^2 (n_x + \kappa) - n_x$, $\mathbf{P}_{j,k|k}$ denotes the j th column of the covariance matrix $\mathbf{P}_{k|k}$, and

the associated weights of sigma points in (9) using the UT are given as

$$W_{j,k} = \begin{cases} \frac{1}{2n_x + 2\lambda}, & j = 1, 2, \dots, 2n_x, \\ \frac{\lambda}{n_x + \lambda}, & j = 2n_x + 1. \end{cases} \quad (10)$$

A. Multi-Source Tracking Filter

The overall posterior density of the ℓ th source tracking filter in (4) is estimated through a set of N_p weighted sigma points via the UKF. In the OMBTLF framework, the estimate of the object state at the current time step k and the predicted observation at the next time step $k + 1$ are computed in three steps:

1) *Prediction Step*: The predictive density $p(\mathbf{x}_k^{(\ell)} | \mathbf{z}_{1:k-1}^{(\ell)})$ for the ℓ th source is characterized by the mean $\hat{\mathbf{x}}_{k|k-1}^{(\ell)}$ and covariance $\mathbf{P}_{k|k-1}^{(\ell)}$, which can be computed as

$$\hat{\mathbf{x}}_{k|k-1}^{(\ell)} = \sum_{j=1}^{N_p} W_{j,k}^{(\ell)} \mathcal{X}_{j,k|k-1}^{(\ell)}, \quad (11)$$

$$\mathbf{P}_{k|k-1}^{(\ell)} = \sum_{j=1}^{N_p} W_{j,k}^{(\ell)} \left[\mathcal{X}_{j,k|k-1}^{(\ell)} - \hat{\mathbf{x}}_{k|k-1}^{(\ell)} \right] \left[\mathcal{X}_{j,k|k-1}^{(\ell)} - \hat{\mathbf{x}}_{k|k-1}^{(\ell)} \right]^T + \mathbf{Q}_v^{(\ell)}, \quad (12)$$

respectively, where the sigma points $\mathcal{X}_{j,k-1|k-1}^{(\ell)}$ that are computed via the posterior of the previous time step are propagated through the nonlinear transition model to generate predicted sigma points defined as $\mathcal{X}_{j,k|k-1}^{(\ell)} = f(\mathcal{X}_{j,k-1|k-1}^{(\ell)})$.

2) *Update Step*: The posterior density of the object state $p(\mathbf{x}_k^{(\ell)} | \mathbf{z}_{1:k}^{(\ell)})$ in (5) for the ℓ th source tracking filter is estimated by updating the computed parameters, $\hat{\mathbf{x}}_{k|k-1}^{(\ell)}$ and $\mathbf{P}_{k|k-1}^{(\ell)}$, of the predictive density in (11) and (12), respectively, via the most recent measurement $\mathbf{z}_k^{(\ell)}$ observed using the ℓ th sensor. The updating procedure is defined as:

$$\hat{\mathbf{x}}_{k|k}^{(\ell)} = \hat{\mathbf{x}}_{k|k-1}^{(\ell)} + \mathbf{K}_k^{(\ell)} (\mathbf{z}_k^{(\ell)} - \hat{\mathbf{z}}_{k|k-1}^{(\ell)}), \quad (13)$$

$$\mathbf{P}_{k|k}^{(\ell)} = \mathbf{P}_{k|k-1}^{(\ell)} - \mathbf{K}_k^{(\ell)} \mathbf{P}_{\mathbf{z},k|k-1}^{(\ell)} (\mathbf{K}_k^{(\ell)})^T, \quad (14)$$

where

$$\hat{\mathbf{z}}_{k|k-1}^{(\ell)} = \sum_{j=1}^{N_p} W_{j,k}^{(\ell)} h(\mathcal{X}_{j,k|k-1}^{(\ell)}), \quad (15)$$

$$\mathbf{K}_k^{(\ell)} = \mathbf{P}_{\mathbf{z},k|k-1}^{(\ell)} (\mathbf{P}_{\mathbf{z},k|k-1}^{(\ell)})^{-1}, \quad (16)$$

$$\mathbf{P}_{\mathbf{z},k|k-1}^{(\ell)} = \sum_{j=1}^{N_p} W_{j,k}^{(\ell)} \left[h(\mathcal{X}_{j,k|k-1}^{(\ell)}) - \hat{\mathbf{z}}_{k|k-1}^{(\ell)} \right] \left[h(\mathcal{X}_{j,k|k-1}^{(\ell)}) - \hat{\mathbf{z}}_{k|k-1}^{(\ell)} \right]^T + \mathbf{Q}_w^{(\ell)}, \quad (17)$$

$$\mathbf{P}_{\mathbf{z},k|k-1}^{(\ell)} = \sum_{j=1}^{N_p} W_{j,k}^{(\ell)} \left[\mathcal{X}_{j,k|k-1}^{(\ell)} - \hat{\mathbf{x}}_{k|k-1}^{(\ell)} \right] \left[h(\mathcal{X}_{j,k|k-1}^{(\ell)}) - \hat{\mathbf{z}}_{k|k-1}^{(\ell)} \right]^T. \quad (18)$$

3) *Predicted Observation Step*: The posterior density of the predicted observation $p(\boldsymbol{\eta}_{k+1}^{(\ell)} | \mathbf{z}_{1:k}^{(\ell)})$ in (6) that is proposed as part of the OMBTLF framework to formulate the significant knowledge gained from the ℓ th source is Gaussian with mean $\hat{\boldsymbol{\eta}}_{k+1|k}^{(\ell)}$ and covariance $\mathbf{Q}_{\boldsymbol{\eta},k+1|k}^{(\ell)}$. These parameters are estimated via weighted sigma points $\mathcal{X}_{j,k|k}^{(\ell)}$ that are drawn according to $\hat{\mathbf{x}}_{k|k}^{(\ell)}$ and $\mathbf{P}_{k|k}^{(\ell)}$ in (13) and (14), respectively, as

$$\mathcal{X}_{j,k+1|k}^{(\ell)} = f\left(\mathcal{X}_{j,k|k}^{(\ell)}\right), \quad (19)$$

$$\hat{\boldsymbol{\eta}}_{k+1|k}^{(\ell)} = \sum_{j=1}^{N_p} W_{j,k}^{(\ell)} h\left(\mathcal{X}_{j,k+1|k}^{(\ell)}\right), \quad (20)$$

$$\mathbf{Q}_{\boldsymbol{\eta},k+1|k}^{(\ell)} = \sum_{j=1}^{N_p} W_{j,k}^{(\ell)} \left[h\left(\mathcal{X}_{j,k+1|k}^{(\ell)}\right) - \hat{\boldsymbol{\eta}}_{k+1|k}^{(\ell)} \right] \left[h\left(\mathcal{X}_{j,k+1|k}^{(\ell)}\right) - \hat{\boldsymbol{\eta}}_{k+1|k}^{(\ell)} \right]^T + \mathbf{Q}_{\mathbf{w}}^{(\ell)}. \quad (21)$$

The estimated mean in (20) and covariance in (21) for each source sensor that characterize the posterior density of the predicted observation are simultaneously and individually transferred to the primary tracking filter. They are then incorporated into the next time step as priors of the filtering process at the primary sensor.

B. Primary Tracking Filter

Given the predicted observation density parameters transferred from each source sensor, defined as $\{\hat{\boldsymbol{\eta}}_{k|k-1}^{(\ell)}, \mathbf{Q}_{\boldsymbol{\eta},k|k-1}^{(\ell)}\}_{\ell=1}^L$, the overall posterior density of the object state $p(\mathbf{x}_k | \mathbf{z}_{1:k}, \mathcal{H}_{2:k})$ in (7) can be equivalently estimated via the UKF as follows.

1) *Prediction Step*: The mean and covariance of the predictive density $p(\mathbf{x}_k | \mathbf{z}_{1:k-1}, \mathcal{H}_{2:k-1})$ in (8) are approximated through a set of weighted sigma points $\mathcal{X}_{j,k-1|k-1}$ that are drawn based on the previous posterior density $p(\mathbf{x}_{k-1} | \mathbf{z}_{1:k-1}, \mathcal{H}_{2:k-1}) = \mathcal{N}(\mathbf{x}_{k-1}; \hat{\mathbf{x}}_{k-1}, \mathbf{P}_{k-1})$ as follows:

$$\hat{\mathbf{x}}_{k|k-1} = \sum_{j=1}^{N_p} W_{j,k} f\left(\mathcal{X}_{j,k-1|k-1}\right), \quad (22)$$

$$\mathbf{P}_{k|k-1} = \sum_{j=1}^{N_p} W_{j,k} \left[f\left(\mathcal{X}_{j,k-1|k-1}\right) - \hat{\mathbf{x}}_{k|k-1} \right] \left[f\left(\mathcal{X}_{j,k-1|k-1}\right) - \hat{\mathbf{x}}_{k|k-1} \right]^T + \mathbf{Q}_{\mathbf{v}}. \quad (23)$$

2) *Update Step*: Once the transferred parameters of L sources that are predicted in the previous time step $k-1$ are received by the primary tracking system, the mean and covariance of the predicted object state density that are estimated in the prediction step are updated based on L predicted observation likelihoods $p(\boldsymbol{\eta}_k^{(\ell)} | \mathbf{x}_k)$ in (8). The procedure for updating the mean and covariance through the set of transferred parameters, defined as $\boldsymbol{\eta}_k^{(\ell)} \sim \mathcal{N}(\hat{\boldsymbol{\eta}}_{k|k-1}^{(\ell)}, \mathbf{Q}_{\boldsymbol{\eta},k|k-1}^{(\ell)})$, where

$\ell = 1, \dots, L$, to estimate the density $p(\mathbf{x}_k | \mathbf{z}_{1:k-1}, \mathcal{H}_{2:k}) = \mathcal{N}(\mathbf{x}_k; \hat{\mathbf{x}}_{k|k-1}^{\boldsymbol{\eta}}, \mathbf{P}_{k|k-1}^{\boldsymbol{\eta}})$ in (8) is given as follows:

$$\hat{\mathbf{x}}_{k|k-1}^{\boldsymbol{\eta}} = \hat{\mathbf{x}}_{k|k-1} + \sum_{\ell=1}^L \mathbf{K}_k^{\boldsymbol{\eta}^{(\ell)}} (\hat{\boldsymbol{\eta}}_{k|k-1}^{(\ell)} - \hat{\boldsymbol{\eta}}_{k|k-1}), \quad (24)$$

$$\mathbf{P}_{k|k-1}^{\boldsymbol{\eta}} = \mathbf{P}_{k|k-1} - \sum_{\ell=1}^L \mathbf{K}_k^{\boldsymbol{\eta}^{(\ell)}} (\mathbf{P}_{\boldsymbol{\eta},k|k-1} + \mathbf{Q}_{\boldsymbol{\eta},k|k-1}^{(\ell)}) (\mathbf{K}_k^{\boldsymbol{\eta}^{(\ell)}})^T, \quad (25)$$

where

$$\mathcal{X}_{j,k|k-1} = f\left(\mathcal{X}_{j,k-1|k-1}\right), \quad (26)$$

$$\hat{\boldsymbol{\eta}}_{k|k-1} = \sum_{j=1}^{N_p} W_{j,k} h\left(\mathcal{X}_{j,k|k-1}\right), \quad (27)$$

$$\mathbf{K}_k^{\boldsymbol{\eta}^{(\ell)}} = \mathbf{P}_{\mathbf{x}\boldsymbol{\eta},k|k-1} (\mathbf{P}_{\boldsymbol{\eta},k|k-1} + \mathbf{Q}_{\boldsymbol{\eta},k|k-1}^{(\ell)})^{-1}, \quad (28)$$

$$\mathbf{P}_{\boldsymbol{\eta},k|k-1} = \sum_{j=1}^{N_p} W_{j,k} \left[h\left(\mathcal{X}_{j,k|k-1}\right) - \hat{\boldsymbol{\eta}}_{k|k-1} \right] \left[h\left(\mathcal{X}_{j,k|k-1}\right) - \hat{\boldsymbol{\eta}}_{k|k-1} \right]^T, \quad (29)$$

$$\mathbf{P}_{\mathbf{x}\boldsymbol{\eta},k|k-1} = \sum_{j=1}^{N_p} W_{j,k} \left[\mathcal{X}_{j,k|k-1} - \hat{\mathbf{x}}_{k|k-1} \right] \left[h\left(\mathcal{X}_{j,k|k-1}\right) - \hat{\boldsymbol{\eta}}_{k|k-1} \right]^T. \quad (30)$$

After updating the mean and covariance in the intermediate step of the OMBTLF framework, a set of sigma points $\mathcal{X}_{j,k|k-1}^{\boldsymbol{\eta}}$ is drawn according to the estimated mean $\hat{\mathbf{x}}_{k|k-1}^{\boldsymbol{\eta}}$ and covariance $\mathbf{P}_{k|k-1}^{\boldsymbol{\eta}}$ in (24) and (25), respectively, to approximate the overall posterior density $p(\mathbf{x}_k | \mathbf{z}_{1:k}, \mathcal{H}_{2:k}) = \mathcal{N}(\mathbf{x}_k; \hat{\mathbf{x}}_{k|k}, \mathbf{P}_{k|k})$ in (7) using the measurement likelihood $p(\mathbf{z}_k | \mathbf{x}_k)$. The mean and covariance parameters that characterize the overall density are estimated, respectively, as follows:

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1}^{\boldsymbol{\eta}} + \mathbf{K}_k (\mathbf{z}_k - \hat{\mathbf{z}}_{k|k-1}), \quad (31)$$

$$\mathbf{P}_{k|k} = \mathbf{P}_{k|k-1}^{\boldsymbol{\eta}} - \mathbf{K}_k \mathbf{P}_{\mathbf{z}\mathbf{z},k|k-1} (\mathbf{K}_k)^T, \quad (32)$$

where

$$\hat{\mathbf{z}}_{k|k-1} = \sum_{j=1}^{N_p} W_{j,k} h\left(\mathcal{X}_{j,k|k-1}^{\boldsymbol{\eta}}\right), \quad (33)$$

$$\mathbf{K}_k = \mathbf{P}_{\mathbf{z}\mathbf{x},k|k-1} (\mathbf{P}_{\mathbf{z}\mathbf{z},k|k-1})^{-1}, \quad (34)$$

$$\mathbf{P}_{\mathbf{z}\mathbf{z},k|k-1} = \sum_{j=1}^{N_p} W_{j,k} \left[h\left(\mathcal{X}_{j,k|k-1}^{\boldsymbol{\eta}}\right) - \hat{\mathbf{z}}_{k|k-1} \right] \left[h\left(\mathcal{X}_{j,k|k-1}^{\boldsymbol{\eta}}\right) - \hat{\mathbf{z}}_{k|k-1} \right]^T + \mathbf{Q}_{\mathbf{w}}, \quad (35)$$

$$\mathbf{P}_{\mathbf{z}\mathbf{x},k|k-1} = \sum_{j=1}^{N_p} W_{j,k} \left[\mathcal{X}_{j,k|k-1}^{\boldsymbol{\eta}} - \hat{\mathbf{x}}_{k|k-1}^{\boldsymbol{\eta}} \right] \left[h\left(\mathcal{X}_{j,k|k-1}^{\boldsymbol{\eta}}\right) - \hat{\mathbf{z}}_{k|k-1} \right]^T. \quad (36)$$

The estimated mean in (31) and the covariance in (32) express the overall density of the object state that is approximated through a set of N_p weighted sigma points by integrating the UKF with OMBTLF to update the predictive density using L predicted observation likelihoods, transferred parameters of multi-source sensors, and the measurement likelihood of the primary sensor. By leveraging transferred knowledge from multi-source sensors, the primary tracking filter is expected to achieve higher estimation accuracy.

V. NUMERICAL RESULTS

We consider an object moving in a maneuvering motion behavior that follows the nonlinear discrete-time dynamic model utilized in [17] defined by

$$\mathbf{x}_k = \begin{bmatrix} 1 & \frac{\sin(\Omega_{k-1}T_s)}{\Omega_{k-1}} & 0 & -\left(\frac{1-\cos(\Omega_{k-1}T_s)}{\Omega_{k-1}}\right) & 0 \\ 0 & \cos(\Omega_{k-1}T_s) & 0 & -\sin(\Omega_{k-1}T_s) & 0 \\ 0 & \frac{1-\cos(\Omega_{k-1}T_s)}{\Omega_{k-1}} & 1 & \frac{\sin(\Omega_{k-1}T_s)}{\Omega_{k-1}} & 0 \\ 0 & \sin(\Omega_{k-1}T_s) & 0 & \cos(\Omega_{k-1}T_s) & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{x}_{k-1} + \mathbf{v}_{k-1}, \quad (37)$$

where the state vector $\mathbf{x}_k = [x_k, \dot{x}_k, y_k, \dot{y}_k, \Omega_k]^T$ consists of the two-dimensional Cartesian coordinate positions, velocities, and a turn rate described as (x_k, y_k) , (\dot{x}_k, \dot{y}_k) , and Ω_k , respectively. The process noise \mathbf{x}_k , which represents the uncertainty in the actual motion model, is assumed to be zero-mean Gaussian noise with covariance

$$\mathbf{Q}_v = \begin{bmatrix} q_1 \frac{T_s^4}{4} & q_1 \frac{T_s^3}{2} & 0 & 0 & 0 \\ q_1 \frac{T_s^3}{2} & q_1 T_s^2 & 0 & 0 & 0 \\ 0 & 0 & q_1 \frac{T_s^4}{4} & q_1 \frac{T_s^3}{2} & 0 \\ 0 & 0 & q_1 \frac{T_s^3}{2} & q_1 T_s^2 & 0 \\ 0 & 0 & 0 & 0 & q_2 T_s \end{bmatrix}. \quad (38)$$

The observed measurements of the l th source and primary sensors follow the measurement models defined in Section III-A. The measurement vector consists of the object range r_k and angle ζ_k :

$$\mathbf{z}_k = [r_k, \zeta_k]^T = \left[\sqrt{x_k^2 + y_k^2}, \arctan(y_k/x_k) \right]^T. \quad (39)$$

The measurement noises of the l th source and the primary sensors are assumed to be zero-mean Gaussian with covariances given by (3), where the noise intensities are denoted $I_w^{(\ell)}$ (for the L multi-sources) and I_w (for the primary) and the common matrix is given by $\mathbf{B}_w = \text{diag}[\sigma_r^2, \sigma_\zeta^2]$.

In the simulation setup, all $L + 1$ sensors (L multi-source sensors and the primary sensor) are impacted by noise processes with the same covariance \mathbf{Q}_v according to our problem formulation. Accordingly, the noise intensity of the L multi-source sensors is assumed to be smaller than that of the primary sensor, i.e., $I_w^{(\ell)} < I_w$. The trajectory of the object under evaluation is identical to the one used in [7]. For comparison purposes, the object state initialization parameters are specified as

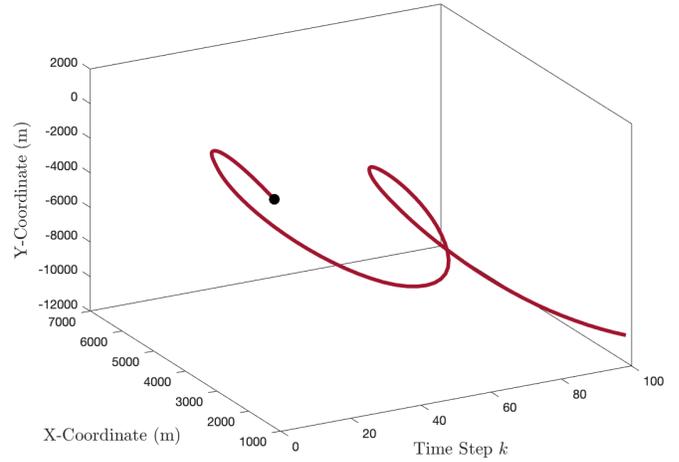


Fig. 2. Trajectory of a moving object in two-dimensional space with \bullet indicating the starting point.

$\mathbf{x}_0 = [1000 \text{ m}, 300 \text{ m/s}, 1000 \text{ m}, 0 \text{ m/s}, -3^\circ/\text{s}]^T$ and $\mathbf{P}_0 = \text{diag}[100 \text{ m}^2, 10 \text{ m}^2/\text{s}^2, 100 \text{ m}^2, 10 \text{ m}^2/\text{s}^2, 100(10^{-3}) \text{ rad}^2/\text{s}^2]$ and the object trajectory has a duration of 100 time steps, as shown in Fig. 2. The simulation parameters are set as $n_x = 5$, $n_z = 2$, $\alpha = 1$, $\kappa = 2$, $T_s = 1 \text{ s}$, $q_1 = 0.1 \text{ m}^2/\text{s}^4$, $q_2 = 1.75 \times 10^{-2} \text{ rad}^2/\text{s}^3$, $\sigma_r = 10 \text{ m}$, and $\sigma_\zeta = \sqrt{10} \times 10^{-3} \text{ rad}$. The root mean square error (RMSE) is an estimation error metric, which is obtained by averaging over 10,000 Monte Carlo (MC) iterations.

TABLE I
SCENARIO 2 SETTINGS

$I_w^{(1)}$	$I_w^{(2)}$	$I_w^{(3)}$	$I_w^{(4)}$	$I_w^{(5)}$	$I_w^{(6)}$	$I_w^{(7)}$	$I_w^{(8)}$	$I_w^{(9)}$	$I_w^{(10)}$
4.25	4.62	1.50	4.65	3.52	1.39	2.11	3.18	4.83	4.85

We consider two scenarios: deterministic and stochastic. In *Scenario 1*, the L multi-source sensors are assumed to experience identical conditions with noise intensity value $I_w^{(\ell)} = 1$. In *Scenario 2*, the noise intensity of the l th source sensor is uniformly drawn within the range of 1 and 5 as $I_w^{(\ell)} \sim \mathcal{U}[1, 5]$, where the corresponding drawn values of the noise intensity for each source sensor are provided in Table I. The RMSE performance of the proposed online multi-source Bayesian transfer learning (OMBTL) with UKF framework has been extensively evaluated for several conditions of the two scenarios to demonstrate the effectiveness of leveraging the knowledge gained from multiple sources.

As shown in Fig. 3, under the primary sensor noise intensity value of $I_w = 8$, the transfer learning unscented Kalman filter (TL-UKF) achieves smaller estimation error by leveraging knowledge from a single sensor and multi-source sensors compared to the isolated UKF (i.e., without transfer learning). Moreover, the TL-UKF with $L = 8$ sources for Scenario 1, dashed line, achieves superior performance compared to the TL-UKF of Scenario 2, dotted line, under the same primary noise intensity and the number of sources. This attribute is

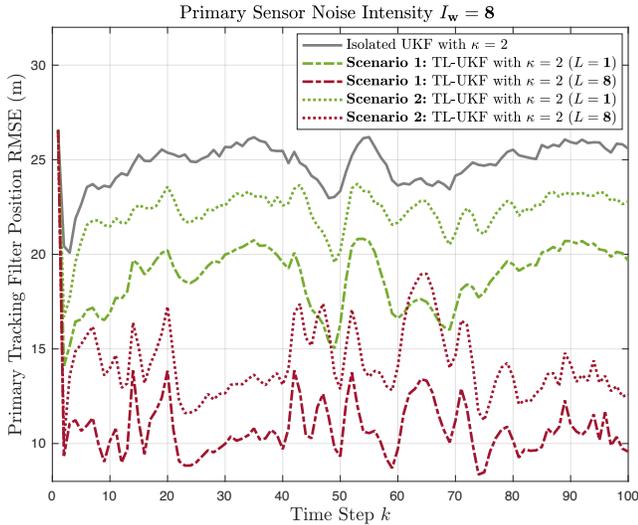


Fig. 3. A comparison of RMSE curves for an isolated UKF and a transfer learning UKF (TL-UKF) using a single sensor ($L = 1$) and $L = 8$ multi-source sensors.

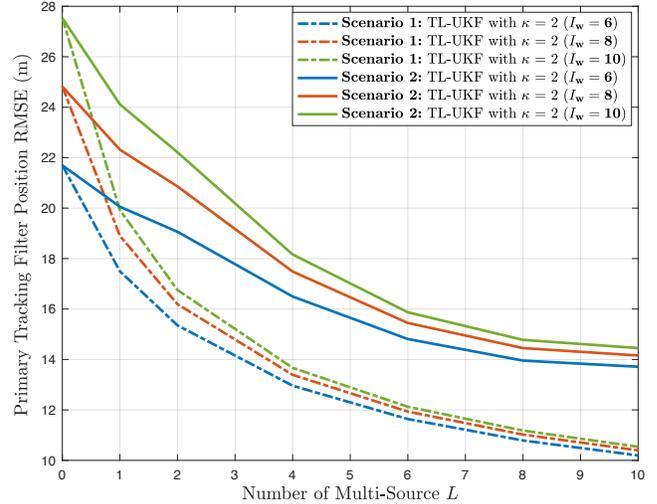


Fig. 4. TL-UKF performance with varying the number of sources L under the primary noise intensity values at $I_w = 6, 8, 10$ for Scenario 1 (dashed) and Scenario 2 (solid).

clearly illustrated in Fig. 4, where the performance of TL-UKF for Scenario 1 surpasses the performance in Scenario 2 under a varying number of multi-sources, L . Note that $L = 0$ represents an isolated system that does not leverage knowledge from other sensors, which explains the identical performances of two scenarios. As expected, leveraging more information from more sources enhances the tracking performance of the primary sensor that experiences higher noise intensity. However, the number of sources is not the only parameter that plays a role in improving the performance of the primary sensor. As shown in Fig. 4, the quality of the transferred knowledge has a significant impact on the performance of the primary sensor, where sources with lower noise intensity, as in Scenario 1, provide useful measurement information compared to sources with slightly higher noise intensity, as in Scenario 2.

VI. CONCLUSION

We developed an online multi-source Bayesian transfer learning (OMBTL) framework that transfers knowledge from L sources to a primary sensor to improve its tracking performance in conjunction with an unscented Kalman filter (UKF). By leveraging predicted observations from $L \geq 2$ sources, the primary sensor, which experiences higher noise intensity, is able to achieve lower estimation error compared to the case of leveraging knowledge from a single source ($L = 1$). A key finding of the proposed framework is that the improvement in primary sensor performance depends proportionally not only on the number of multi-sources but also on the quality of the predicted observation information transferred by the sources.

REFERENCES

- [1] A. Karbalayghareh, X. Qian, and E. R. Dougherty, "Optimal Bayesian transfer learning," *IEEE Trans. Signal Process.*, vol. 66, no. 14, pp. 3724–3739, 2018.
- [2] S. J. Pan and Q. Yang, "A survey on transfer learning," *IEEE Trans. Knowl. Data Eng.*, vol. 22, no. 10, pp. 1345–1359, 2009.

- [3] C. Foley and A. Quinn, "Fully probabilistic design for knowledge transfer in a pair of Kalman filters," *IEEE Signal Process. Lett.*, vol. 25, no. 4, pp. 487–490, 2017.
- [4] R. E. Kalman, "A new approach to linear filtering and prediction problems," *J. Basic Eng.*, vol. 82, no. 1, pp. 35–45, 1960.
- [5] S. Julier, J. Uhlmann, and H. F. Durrant-Whyte, "A new method for the nonlinear transformation of means and covariances in filters and estimators," *IEEE Trans. Autom. Control*, vol. 45, no. 3, pp. 477–482, March 2000.
- [6] I. Arasaratnam and S. Haykin, "Cubature Kalman filters," *IEEE Trans. Autom. Control*, vol. 54, no. 6, pp. 1254–1269, 2009.
- [7] O. Alotaibi and B. L. Mark, "Object tracking incorporating transfer learning into an unscented Kalman filter," in *Proc. 58th Annu. Conf. Inf. Sci. Syst. (CISS)*, Princeton, NJ, March 2024, pp. 1–6.
- [8] —, "Object tracking incorporating transfer learning into unscented and cubature Kalman filters," *arXiv:2408.07157*, 2024.
- [9] J. Lu, H. Zuo, and G. Zhang, "Fuzzy multiple-source transfer learning," *IEEE Trans. Fuzzy Syst.*, vol. 28, no. 12, pp. 3418–3431, 2020.
- [10] K. Li, J. Lu, H. Zuo, and G. Zhang, "Multi-source contribution learning for domain adaptation," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 33, no. 10, pp. 5293–5307, 2022.
- [11] Q. Wu, H. Wu, X. Zhou, M. Tan, Y. Xu, Y. Yan, and T. Hao, "Online transfer learning with multiple homogeneous or heterogeneous sources," *IEEE Trans. Knowl. Data Eng.*, vol. 29, no. 7, pp. 1494–1507, 2017.
- [12] B. Yang, J. Zhu, Z. Yu, F. Fan, X. Liu, and R. Ni, "Fast adaptation trajectory prediction method based on online multisource transfer learning," *IEEE Trans. Autom. Sci. Eng.*, pp. 1–16, 2024.
- [13] R. Olfati-Saber, "Distributed Kalman filter with embedded consensus filters," in *Proc. 44th IEEE Conf. Decis. Control*, 2005, pp. 8179–8184.
- [14] —, "Distributed Kalman filtering for sensor networks," in *Proc. 46th IEEE Conf. Decis. Control*, 2007, pp. 5492–5498.
- [15] Y. Ho and R. Lee, "A Bayesian approach to problems in stochastic estimation and control," *IEEE Trans. Autom. Control*, vol. 9, no. 4, pp. 333–339, October 1964.
- [16] S. J. Julier and J. K. Uhlmann, "A new extension of the Kalman filter to nonlinear systems," in *Proc. Signal Process., Sensor Fusion, Target Recognit. VI*, vol. 3068, 1997, pp. 182–193.
- [17] Y. Bar-Shalom, X. R. Li, and T. Kirubarajan, *Estimation with Applications to Tracking and Navigation: Theory Algorithms and Software*. New York, NY: Wiley, 2001.